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Notices

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Edited by J. H. Curtiss

VOLUME 5, NUMBER 2 ISSUE NO. 30 APRIL 1958

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MEETINGS

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Please send in abstracts of papers to be presented in person well in advance of the deadline.

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
547	June 20, 1958	Corvallis, Oregon	May 7
548	August 25-30, 1958 (63rd Summer Meeting)	Cambridge, Massachusetts	July 11
549	October 25, 1958	Princeton, New Jersey	Sept. 11
550	November 21-22, 1958	Pomona, California	Oct. 8
551	November 28-29, 1958	Durham, North Carolina	Oct. 8
552	November 28-29, 1958	Evanston, Illinois	Oct. 8
553	January 20-22, 1959 (65th Annual Meeting)	Philadelphia, Pennsylvania	Dec. 5
	Summer Meeting, 1959	Salt Lake City, Utah	
	November, 1959	Detroit, Michigan	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

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FIVE HUNDRED FORTY-FOURTH MEETING

Chicago, Illinois
April 18-19, 1958

PROGRAM

The five hundred forty-fourth meeting of the American Mathematical Society will be held at the University of Chicago, Chicago, Illinois, on Friday and Saturday, April 18-19, 1958. All sessions will be held in Eckhart Hall.

Registration will be in the Common Room on the second floor of Eckhart Hall, beginning at 9:00 A.M., Friday.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor George Whaples of Indiana University will address the Society on the topic "Quasi Galois fields and local class field theory." Professor Whaples' lecture will be held in Room 133 at 2:00 P.M. on Friday, April 18.

Sessions for the presentation of contributed papers will be held at 10:00 A.M. on Friday and Saturday, and 3:15 P.M. on Friday.

If necessary there will be a special session Saturday afternoon for the presentation of papers which failed to meet the deadline. Further details will be available at the registration desk.

There will be a tea in the Common Room of Eckhart Hall starting at 4:15 P.M. on Friday.

The facilities of Hutchinson Commons, a dining hall directly across from Eckhart Hall, will be available to members of the Society and guests for all meals.

The following hotels have agreed to accommodate those members of the Society making reservations in advance:

In the University district

	Single	Double
Shoreland Hotel 5454 South Shore Drive	$7.00-$10.00	$10.00-$15.00
Del Prado Hotel 5307 South Hyde Park Blvd.	7.00 up	10.00 up
Hotels Windermere 1642 East 56th Street	7.50-14.00	11.00-14.00
Hotel Broadview 5400 South Hyde Park Blvd.	4.00-6.00	6.00-8.00

Hotel Miramar 6218 South Woodlawn	$4.50	$7.00
Hyde Park Y.M.C.A. 1400 East 53rd Street	2.25	

In the Loop district

The Conrad Hilton	$7.00 up	$12.50 up

Reservations should be made directly with the hotel.

Abstracts of the papers which are to be presented in person at this meeting appear on pages 204-222 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program below is followed by (544-20) indicating that the abstract can be found under the designation 544-20 among the published abstracts.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

FRIDAY, 10:00 A.M.

Session on Analysis, Room 133

(1) Numerical range and Banach space geometry. Preliminary report
Mr. Günter Lumer, University of Chicago (544-20)

(2) Mean functions of almost periodic functions on a group. Preliminary report
Professor C. E. Langenhop, Iowa State College of Agriculture and Mechanic Arts (544-36)

(3) On a class of non-continuable analytic functions
Mr. F. W. Carroll, Jr., Purdue University (544-21)

(4) On minimal norms
Dr. A. S. Householder, Oak Ridge National Laboratory (544-6)

(5) Inversion of a class of convolutions
Professor Harry Pollard and Dr. Harold Widom, Cornell University (544-23)

(6) Bounded sequences summed by matrices
Professor G. M. Petersen, University of New Mexico (544-24)

Session on Algebra, Room 202

(7) Injective modules
Mr. Eben Matlis, University of Chicago (544-1)

(8) On unique factorization in semigroups of the integers
Mr. Newcomb Greenleaf and Professor R. J. Wisner, Haverford College (544-37)

(9) On the probability of relatively prime values of a function
Professor Paul Erdős, University of Toronto, and Professor G. G. Lorentz, Wayne State University (544-25)

(10) Algebraic sets and finitely generated semigroups
Dr. E. C. Posner, University of Wisconsin (544-31)

(11) On the H_p-subgroup of a group. Preliminary report
Professor D.R. Hughes, Ohio State University (544-5)

(12) On nilstable algebras
Professor L. A. Kokoris, Washington University (544-13)

(13) On the group of the composition of two graphs
Professor Frank Harary, University of Michigan and Institute for Advanced Study (544-3)

FRIDAY, 2:00 P.M.

General Session, Room 133

Quasi Galois fields and local class field theory (One hour)
Professor George Whaples, Indiana University

FRIDAY, 3:15 P.M.

Session on Applied Mathematics, Room 133

(14) Convergence of a two-point method for the numerical solution of sets of simultaneous equations
Dr. W. M. Kincaid, University of Michigan (544-26)

(15) Numerical solution of algebraic equations
Dr. W. C. Taylor, Aberdeen Proving Ground (544-34)

(16) Formal solutions and a related equation for a class of differential equations of a hydrodynamic type
Professor R. E. Langer, University of Wisconsin (544-9)

(17) Orderings of the successive overrelaxation method
Dr. R. S. Varga, Westinghouse Electric Corporation (544-12)

(18) On the multigroup diffusion model
Dr. M. A. Martino, Jr. and Dr. G. J. Habetler, General Electric Company, Schenectady, New York (544-28)

(19) Thermal shock on the surface of the elastic half-space
Dr. J. L. Bailey, Michigan State University (544-11)

(20) Damped free vibrations with a finite number of oscillations
Professor Oswald Wyler, University of New Mexico (544-16)

Session on Topology, Room 202

(21) Extending homeomorphisms on the pseudo-arc
Dr. G. R. Lehner, University of Wisconsin, Milwaukee (544-19)

(22) On cluster sets in n-dimensional space
Mr. P. T. Church, University of Michigan (544-7)

(23) Contractibility of spheres in spaces of homeomorphisms
Professor J. H. Roberts, Duke University (544-22)

(24) Concerning an unnecessary condition in certain upper semicontinuous decompositions of E^3 into itself
Professor L. F. McAuley, University of Wisconsin (544-32)

(25) A summability theorem in countable toral groups
Professor Mark Mahowald, Xavier University (544-42)

(26) Fixed points of a circle group acting on a cohomology n-sphere. Preliminary report
Mr. R. B. Paine, Central Michigan College (544-15)

(27) Stable Postnikov invariants
Mr. E. L. Lima, University of Chicago (544-27)

SATURDAY, 10:00 A.M.

Session on Topology, Room 133

(28) Criteria for immersibility of manifolds
Mr. M. W. Hirsch, University of Chicago (544-33)

(29) Differentiation on manifolds without a connection
Mr. B. L. Foster, University of Wisconsin

(30) A class of discrete uniform subgroups of solvable Lie groups
Professor Louis Auslander, Indiana University

(31) Analytic equivalence of curves
Professor A. H. Wallace, University of Toronto (544-4)

(32) On continuous associative multiplications in arcwise connected compact Hausdorff spaces
Professor S. T. Hu, Wayne State University (544-40)

(33) A theorem on semi-simplicial monoid complexes
Dr. Dieter Puppe, Institute for Advanced Study (544-30)

(34) Comparison of singular and Cech homology in locally connected spaces
Dr. Sibe Mardešić, Institute for Advanced Study (544-14)

Session on Logic and Statistics, Room 206

(35) On the interdependence of the Kolmogorov differential equation
Professor D. G. Austin, Yale University (544-39)

(36) Statistical metric spaces arising from sets of random variables
Professor Berthold Schweizer, San Diego State College,

and Professor Abe Sklar, Illinois Institute of Technology (544-38)

(37) On semi-groups of positive matrices. II
Professor W. B. Jurkat, Syracuse University (544-41)

(38) Definability in polynomial rings
Mr. D. S. Scott, Princeton University (544-44)

(39) Remarks on isolic arithmetic
Dr. Anil Nerode, Institute for Advanced Study (544-29)

Session on Analysis, Room 202

(40) On almost periodic solutions of a class of differential equations
Mr. G. H. Meisters, Iowa State College of Agriculture and Mechanic Arts
(Introduced by Professor C. E. Langenhop) (544-35)

(41) Research into the sets of solutions of certain systems of nonlinear partial differential equations of higher order
Mr. Johann Martinek and Dr. Henry de Beaumont, Reed Research, Inc. (544-17)

(42) Solution on nonlinear differential equations with a parameter by asymptotic series
Professor W. R. Wasow, University of Wisconsin (544-2)

(43) Proof of the fundamental theorem on implicit functions by composite gradient corrections
Professor W. L. Hart, University of Minnesota, and
Professor T. S. Motzkin, University of California, Los Angeles (544-8)

(44) Generalizations of the Liouville theorem to higher dimensions
Professor G. S. Young, University of Michigan (544-10)

(45) On the Lebesgue constants for Laplace series
Professor Lee Lorch, Philander Smith College (544-18)

SUPPLEMENTARY PROGRAM

(To be presented by title)

(46) Horn sentences in identity theory
Mr. K. I. Appel, University of Michigan

(47) Factorization in group algebras
Mr. P. J. Cohen, University of Rochester

(48) Concerning a property of continued fractions
Professor D. F. Dawson, University of Missouri

(49) The group of bilinear transformations of a quadratic extension Γ of a field K
Professor John De Cicco, De Paul University

(50) On endomorphisms of nonabelian groups
Professor W. E. Deskins, Michigan State University

(51) An extension of a theorem of A. Dold and R. Thom
Professor Edward Fadell, University of Wisconsin

(52) Uniqueness theorems for singular Cauchy problems. Preliminary report
Mr. D. W. Fox, University of Maryland

(53) A discontinuous eigenfunction problem. Preliminary report
Professor Morris Marden, University of Wisconsin, Milwaukee

(54) An extension of a spectral theorem of M. A. Naimark
Dr. M. A. Martino, General Electric Company, Schenectady, New York

(55) Mapping and space relations
Mr. P. E. McDougle, University of Virginia

(56) Semi-compact mappings
Mr. P. E. McDougle, University of Virginia

(57) Some stone spaces and recursion theory. I
Dr. Anil Nerode, Institute for Advanced Study

(58) Two theorems on (C,1) summability
Dr. E. C. Posner, University of Wisconsin

(59) Random walk on monoids
Dr. M. P. Schützenberger, Paris, France

(60) Homomorphisms of a monoid onto a group
Dr. M. P. Schützenberger, Paris, France

(61) Some results on preference patterns. Preliminary report
Mr. Richard Stearns, Carleton College
(Introduced by: Professor F. L. Wolf)

(62) The structure of unitary and orthogonal quaternion matrices
Professor N. A. Wiegmann, Catholic University of America

(63) Ideal points and order axioms in an incidence geometry
Professor Oswald Wyler, University of New Mexico

J. W. T. Youngs
Associate Secretary

Bloomington, Indiana
March 5, 1958

FIVE HUNDRED FORTY-FIFTH MEETING

Stanford, California
April 18-19, 1958

PROGRAM

The five hundred forty-fifth meeting of the American Mathematical Society will be held on Friday and Saturday, April 18-19, 1958, at Stanford University, California. (The morning and afternoon sessions on Friday will be at the Stanford Research Institute in Menlo Park, and the remaining sessions on the Stanford campus.)

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor W. W. Rogosinski of the University of Colorado and the University of Durham will address the Society Saturday afternoon on "General moment problems". His talk will be at 1:00 P. M. in Physics 100p.

By invitation of the same Committee, a symposium will be held Friday on Banach Algebras and Harmonic Analysis. Sessions will be held on Friday morning and afternoon at the S. R. I. auditorium, and Friday evening on the campus in Physics 100p. The addresses on Friday morning (starting at 10:00 A.M.) are by Professor Richard Arens, University of California, Los Angeles -- "Some topics in the theory of commutative Banach algebras", and Professor Arne Beurling, Institute for Advanced Study -- "Dirichlet spaces". The afternoon and evening sessions (1:45 P.M. and 8:00 P.M.) will be devoted to shorter addresses and perhaps a panel discussion, the participants being Professors Paul Civin, University of Oregon, Karel deLeeuw, Stanford University, H. A. Dye, University of Southern California, Michael Fell, University of Washington, N. J. Fine, University of Pennsylvania, Henry Helson, University of California, Berkeley, Edwin Hewitt, University of Washington, I. I. Hirschman, Jr., Washington University, K. M. Hoffman, Massachusetts Institute of Technology, Harry Pollard, Cornell University, H. L. Royden, Stanford University, and Walter Rudin, University of Rochester. (The organizing committee for the symposium consisted of Professors Hewitt (Chairman), Dye, Helson, and Hirschman.) Full details of the afternoon and evening sessions of the symposium will be available at its morning session, and may be obtained in advance by writing to V. L. Klee, Department of Mathematics, University of Washington, Seattle 5, Washington.

Sessions for contributed papers will be held at 9:15 A.M. and 2:15 P.M. Saturday in Engineering 276 and 283, to be followed by a tea at the Bowman Alumni House. Late papers can be accommodated if necessary, details to be available at the Registration Desk. Registration on Friday morning and afternoon will be at the S. R. I. auditorium, and on Saturday morning and afternoon outside Room 70, the Mathematics Office on the Inner-Quad. The Bowman Alumni House will be available all day Saturday for informal gatherings of those in attendance.

Stanford University adjoins the city of Palo Alto, which is served by Southern Pacific Railway and Greyhound Bus. The Stanford Research Institute is located in Menlo Park, two to three miles from the University and between El Camino Real (U.S. 101) and Middlefield Road. S. R. I. can be reached by Greyhound from Palo Alto and also from the San Francisco Airport.

There are numerous excellent motels within easy driving distance of the University. (A detailed list may be obtained from the Chamber of Commerce, 725 University Avenue, Palo Alto, California.) For those who travel to the meeting by means of public transportation, the following hotels are recommended: President Hotel, Palo Alto; Cardinal Hotel, Palo Alto. Bus service is available between these hotels and the University.

Abstracts of the papers which are to be presented in person at this meeting appear on pages 223-234 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program below is followed by (545-14), indicating that the abstract can be found under the designation 545-14 among the published abstracts.

PROGRAM OF THE SYMPOSIUM
BANACH ALGEBRAS AND HARMONIC ANALYSIS

FRIDAY, 10:00 A.M.

Session I, Auditorium, Stanford Research Institute

Professor Richard Arens, University of California, Los Angeles
Some topics in the theory of commutative Banach algebras
Professor Arne Beurling, Institute for Advanced Study
Dirichlet spaces

FRIDAY, 1:45 P.M.

Session II, Auditorium, Stanford Research Institute

Shorter addresses*

FRIDAY, 8:00 P.M.

Session III, Physics 100p, Stanford campus

Shorter addresses and panel discussion*

*Speakers are Professors Civin, deLeeuw, Dye, Fell, Fine, Helson, Hewitt, Hirschman, Hoffman, Pollard, Royden, and Rudin. Full details of the afternoon and evening sessions will be available at the morning session.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

SATURDAY, 9:15 A.M.

Session on Geometry and Topology, Engineering 276, Stanford Campus

(1) On n-regular curves
Dr. Lida K. Barrett, University of Utah (545-14)

(2) Homogeneous continua which are circularly chainable and contain an arc
Professor C. E. Burgess, University of Utah (545-16)

(3) Homomorphisms of rings of continuous functions
Professor Leonard Gillman and Professor Meyer Jerison, Purdue University (545-15)

(4) Convex structures and continuous selections
Professor E. A. Michael, University of Washington

(5) Graphs with transitively acting groups
Professor G. O. Sabidussi, Tulane University (545-27)

(6) Transient flows in networks
Professor David Gale, Brown University and RAND Corporation (545-8)

(7) On triangle inequalities for statistical metric spaces
Professor Berthold Schweizer, San Diego State College, and Professor Abe Sklar, Illinois Institute of Technology (545-20)

(8) On piece-wise congruence. Preliminary report
Dr. Albert Cahn, RAND Corporation, and Professor E. G. Straus, University of California, Los Angeles

(9) Some remarks on complex n-plane bundles
Dr. F. P. Peterson, Princeton University (545-2)

Session on Number Theory, Analysis, and Applied Mathematics

(10) Asymptotic properties of the sum of the divisors of an integer
Professor E. E. Kohlbecker, University of Utah (545-12)

(11) Arithmetical properties of Bernoulli polynomials
Professor Morgan Ward, California Institute of Technology (545-18)

(12) Pansions, Fourier transforms, and boundary value problems
Professor Jacob Korevaar, University of Wisconsin (545-24)

(13) Fourier series approximation to nonlinear parabolic boundary value problems
    Dr. J. B. Rosen, Shell Development Company, Emeryville, California (545-3)

(14) A Prüfer transformation for differential systems
    Professor W. T. Reid, University of California, Los Angeles and Northwestern University (545-26)

(15) Asymptotic distribution of the eigenvalues for the lower part of the one-dimensional Schrödinger operator spectrum
    Mr. C. W. Clark, University of Washington (545-11)

(16) Explicit perturbation formulae and convergence estimates. Preliminary report
    Professor F. H. Brownell, University of Washington

(17) Elliptic forms in Hilbert space, and second order elliptic differential equations
    Professor R. F. Dennemeyer, Long Beach State College (545-4)

(18) Approximate solutions of parabolic equations
    Mr. Milton Lees, Shell Development Company, Emeryville, California (545-25)

SATURDAY, 1:00 P.M.

General session, Physics 100p, Stanford campus

General moment problems (One hour)
    Professor W. W. Rogosinski, University of Colorado and University of Durham

SATURDAY, 2:15 P.M.

Session on Functional Analysis, Engineering 276, Stanford campus

(19) Automorphic group representations
    Dr. R. J. Blattner, University of California, Los Angeles (545-10)

(20) On some Banach algebras of Wermer
    Dr. K. M. Hoffman and Professor I. M. Singer, Massachusetts Institute of Technology (545-23)

(21) Šilov boundaries induced by certain Banach algebras
    Mr. W. W. Comfort, University of Washington (545-9)

(22) Some subreflexive Banach spaces
    Mr. R. R. Phelps, University of Washington

(23) Remarks on the geometry of normed linear spaces
    Professor V. L. Klee, Jr., University of Washington (545-21)

(24) The norm of a real linear transformation in Minkowski space

Professor A. E. Taylor, University of California, Los Angeles (545-7)

(25) Linear operators and their conjugates
Mr. Seymour Goldberg, University of California, Los Angeles (545-6)

(26) Generalized resolvents of a symmetric operator
Professor R. C. Gilbert, University of California, Riverside (545-13)

Session on Applied Mathematics and Probability, Engineering 283, Stanford campus

(27) Divergence from Bessel's integral in a radiation equation
Professor V. W. Bolie, Iowa State College of Agriculture and Mechanic Arts (545-1)

(28) Stability of a numerical solution of differential equations
Professor W. E. Milne, U. S. Naval Postgraduate School, and Mr. R. R. Reynolds, Oregon State College (545-22)

(29) On hydrodynamic stability of a wake
Professor L. N. Howard, Massachusetts Institute of Technology (545-17)

(30) Natural sorting
Professor R. M. Baer, Cal Research Corporation, Berkeley, California, and Professor Paul Brock, University of Michigan (545-5)

(31) On the structure of stochastic independence
Professor C. B. Bell, Jr., Xavier University of Louisiana and Stanford University

(32) Symmetrizable Markov matrices
Dr. H. P. Kramer, Bell Telephone Laboratories, Murray Hill, New Jersey (545-19)

(33) Toeplitz matrices and the absorption problem
Professor F. L. Spitzer, California Institute of Technology (545-28)

SUPPLEMENTARY PROGRAM

(To be presented by title)

(34) Asymptotic distribution of the eigenvalues for the lower part of the n-dimensional Schrődinger operator spectrum
Mr. C. W. Clark, University of Washington

(35) Twelve points in PG(5,3) with 95040 self-transformations
Professor H. S. M. Coxeter, University of Toronto

(36) Linear completeness and hyperbolic trigonometry
Professor C. M. Fulton, University of California, Davis

(37) On the numerical solution of three dimensional Dirichlet problems
Professor Donald Greenspan, Purdue University
(38) Eigenvalues of the unitary part of a matrix
Professor Alfred Horn and Professor Robert Steinberg, University of California, Los Angeles
(39) Semimartingales going down
Dr. Klaus Krickeberg, University of Wurzburg
(40) Incomparable theories
Professor Richard Montague, University of California, Los Angeles
(Introduced by Professor J. D. Swift)

V. L. Klee, Jr.
Associate Secretary

Seattle, Washington
March 6, 1958

FIVE HUNDRED FORTY-SIXTH MEETING

New York, New York
April 24-26, 1958

PROGRAM

The five hundred forty-sixth meeting of the American Mathematical Society will be held at Columbia University in New York City on Thursday, Friday, and Saturday, April 24-26, 1958.

A Symposium on Combinatorial Designs and Analysis (sponsored by the Society with the aid of the Office of Ordnance Research) will be held in conjunction with the regular meeting. There will be four sessions of the Symposium, beginning at 10:00 A.M. and 2:00 P.M. on Thursday, 10:00 A.M. on Friday, and 10:00 A.M. on Saturday. All sessions of the Symposium will be held in the McMillin Academic Theatre.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor I. M. Singer of the Massachusetts Institute of Technology will address the Society on Friday at 2:00 P.M. on "Connections and holonomy groups", and Professor J. C. Moore of Princeton University will deliver an hour address entitled "A survey of some modern developments in homotopy theory" on Saturday at 2:00 P.M. Both hour addresses will be given in the Pupin Physical Laboratories, Room 301.

There will be sessions for contributed papers at 3:15 P.M. on Friday, and at 10:00 A.M. and 3:15 P.M. on Saturday.

The Employment Register will be maintained in Pegram Physical Laboratories, Room 401, on Friday afternoon and Saturday (the exact times to be announced at the meeting).

Room 401, Pegram Physical Laboratories, will also be available as a general lounge from 2:00 P.M. to 5:00 P.M. on Friday, and from 9:00 A.M. to 5:00 P.M. on Saturday.

The Council of the Society will meet at 5:00 P.M. on Friday in the Faculty Club.

Columbia University may be reached by the Broadway - 7th Avenue line of the IRT Subway at the 116th Street station. The McMillin Academic Theatre is located at the north corner of 116th Street and Broadway. The Pupin Physical Laboratories are on the south side of 120th Street at Broadway. Pegram is a new building connected to Pupin.

A registration desk will be located in the McMillin Academic Theatre on Thursday and on Friday morning. It will be moved to the third floor of the Pupin Physical Laboratories on Friday afternoon and Saturday.

Mail and telegrams intended for those attending the Symposium and/or meeting should be addressed as follows: American Mathematical Society, c/o General Delivery Local Mail Department, Room 112 Low Library, Columbia University, New York 27, New York. This service will not be available after noon on Saturday.

Abstracts of the papers which are to be presented in person at this meeting appear on pages 235-252 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program below is followed by (546-33), indicating that the abstract can be found under the designation 546-33 among the published abstracts.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

THURSDAY, 10:00 A.M.

Symposium Session I, Existence and construction of combinatorial designs, McMillin Academic Theatre

Current studies on combinatorial designs (45 minutes)
Professor Marshall Hall, Jr., Ohio State University

Quadratic extensions of cyclic planes
Professor R. H. Bruck, University of Wisconsin

Homomorphisms of projective planes
Professor D. R. Hughes, Ohio State University

The cyclotomic number of order 10
Professor A. L. Whiteman, University of Southern California

Finite division algebras and finite planes
Professor A. A. Albert, University of Chicago

THURSDAY, 2:00 P.M.

Symposium Session II, Combinatorial analysis of discrete extremal problems, McMillin Academic Theatre

Some recent applications of the theory of linear inequalities to extremal combinatorial analysis (45 minutes)
Dr. A. J. Hoffman, General Electric Company

Compound and induced matrices in combinatorial analysis
Professor H. J. Ryser, Ohio State University

Duality structure
Professor A. W. Tucker, Princeton University

Linear inequalities and the Pauli principle
Professor H. W. Kuhn, Bryn Mawr College

On some communication network problems
Dr. R. E. Kalaba, RAND Corporation

Permanents of doubly stochastic matrices
Dr. Morris Newman, National Bureau of Standards

FRIDAY, 10:00 A.M.

Symposium Session III, Problems of communications, transportation and logistics, McMillin Academic Theatre

Dynamic programming and combinatorial processes (45 minutes)
Dr. R. E. Bellman, RAND Corporation

A problem in binary encoding
Dr. E. N. Gilbert, Bell Telephone Laboratories

Directed graphs and assembly schedules
Dr. J. Foulkes, Bell Telephone Laboratories

Solutions of large scale transportation problems
Professor Murray Gerstenhaber, University of Pennsylvania and Institute for Advanced Study

Discussion will be led by Dr. M. M. Flood of the Engineering Research Institute, University of Michigan.

FRIDAY, 2:00 P.M.

General Session, Pupin Physical Laboratories, Room 301

Connections and holonomy groups (One hour)
Professor I. M. Singer, Massachusetts Institute of Technology

FRIDAY, 3:15 P.M.

Session on Analysis, Pupin Physical Laboratories, Room 301

(1) Linear extremal problems for polynomials
Dr. H. S. Shapiro, New York University (546-33)

(2) On the method of minimum deviation
Professor H. W. E. Schwerdtfeger, McGill University (546-19)

(3) New results in the theory and techniques of Chebyshev fitting
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York (546-34)

(4) Some identities and inequalities involving ultraspherical polynomials
Dr. A. E. Danese, Eastman Kodak Company, Rochester, New York (546-27)

Note: Members are requested to leave this room before 4:15 P.M. since the room will be used for another meeting.

Session on Statistics and Probability, Logic and Foundations, Pupin Physical Laboratories, Room 428

(5) Characterization and decomposition of stochastic processes with stationary independent increments
Dr. V. E. Beneš, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (546-29)

(6) Markov operators and their associated semi-groups
Professor R. K. Getoor, University of Washington (546-9)

(7) Representation of sequential circuits in combinatory logic
Professor F. B. Fitch, Yale University (546-30)

(8) Constructive ordinals in formal systems. Preliminary report
Professor J. R. Shoenfield, Duke University (546-35)

(9) Unified formulation of logical systems
Mr. D. E. Schroer, University of Rochester (546-41)

(10) A connection between many-valued logics and the two-valued system
Professor L. W. Small, Yonkers, New York (546-8)

SATURDAY, 10:00 A.M.

Symposium Session IV, Numerical analysis of discrete problems, McMillin Academic Theatre

Teaching combinatorial tricks to a computer (45 minutes)
Professor D. H. Lehmer, University of California, Berkeley

The computational size of the ten by ten orthogonal Latin square problem
Professor C. B. Tompkins and Professor L. J. Paige, University of California, Los Angeles

Some discrete variable computations
Professor John Todd and Dr. Olga Taussky, California Institute of Technology

Some combinatorial problems associated with finite partially ordered sets
Professor R. P. Dilworth, California Institute of Technology

An enumeration technique for a class of combinatorial problems
Professor R. J. Walker, Cornell University

Two methods of search in an n-cube
Professor A. M. Gleason, Harvard University

Isomorph rejection in exhaustive search techniques
Professor J. D. Swift, University of California, Los Angeles

Session on Analysis, Pupin Physical Laboratories, Room 301

(11) Fractional integration and chain transforms
Professor Charles Fox, McGill University (546-28)

(12) Linear operator equations
Mr. Günter Lumer, University of Chicago, and Professor Marvin Rosenblum, University of Virginia (546-20)

(13) A sufficient condition for an integral operator to have a trace
Dr. W. F. Stinespring, Institute for Advanced Study (546-43)

(14) The finite convolution transform
Professor Jerome Blackman, Syracuse University, and Professor Harry Pollard, Cornell University (546-3)

(15) Some unusual aspects of harmonic analysis on the group of 2x2 real unimodular matrices
Dr. R. A. Kunze and Dr. E. M. Stein, Massachusetts Institute of Technology (546-39)

(16) Integrals over function space. III
Professor W. F. Eberlein, University of Rochester (546-38)

(17) On Green's theorem
Mr. P. J. Cohen, University of Rochester (546-37)

(18) On the Hilbert matrix. II
Professor Marvin Rosenblum, University of Virginia (546-25)

(19) Remarks on (m,n)-compact spaces
Professor I. S. Gál, Cornell University (546-36)

Session on Geometry and Topology, Pupin Physical Laboratories, Room 428

(20) Pencils of null systems. II
Professor Seymour Schuster, Polytechnic Institute of Brooklyn (546-10)

(21) On the osculating spaces to analytic projective curves
Reverend F. A. Homann, University of Pennsylvania (546-5)

(22) The set of non-linearity of a convex piecewise-linear function
Dr. Chandler Davis, Institute for Advanced Study (546-15)

(23) Contact manifolds

Dr. J. W. Gray, Institute for Advanced Study (546-32)

(24) Calculation of conformal parameters of imbedded Riemann surfaces

Mr. A. M. Garsia, Massachusetts Institute of Technology (546-16)

(25) A general integral geometric formula, with application to curvatures. Preliminary report

Professor Herbert Federer, Brown University (546-1)

(26) The cartesian product of a certain nonmanifold and a line is E^4 .

Professor R. H. Bing, University of Wisconsin and Institute for Advanced Study (546-22)

(27) The "space of loops" isomorphism

Mr. J. D. Stasheff, Princeton University (546-42)

(28) Fixed points for special transformations of plane continua

Professor O. H. Hamilton, Oklahoma State University (546-12)

SATURDAY, 2:00 P.M.

General Session, Pupin Physical Laboratories, Room 301

A survey of some modern developments in homotopy theory (One hour)

Professor J. C. Moore, Princeton University

SATURDAY, 3:15 P.M.

Session on Analysis, Pupin Physical Laboratories, Room 301

(29) A theorem on L^p Fourier series

Professor R. P. Gosselin, University of Connecticut (546-17)

(30) Functions which have generalized Riemann derivatives

Dr. Costas Kassimatis, Cornell University (546-6)

(31) The Cauchy problem for the iterated wave equation

Professor E. P. Miles, Jr., University of Maryland and Alabama Polytechnic Institute (546-26)

(32) On discontinuous initial value problems for certain nonlinear partial differential equations

Professor Avron Douglis, University of Maryland

(33) Poincaré's perturbation method and topological degree

Professor Jane Cronin Scanlon, Polytechnic Institute of Brooklyn (546-14)

(34) A note on the solution of the differential equation of the type $g(x,y,y') = 0$

Professor Smbat Abian and Professor A. B. Brown, Queens College (546-24)

(35) On the asymptotic behavior of the integral curves of a certain non-linear differential equation
Dr. Abolghassem Ghaffari, National Bureau of Standards, Washington, D. C. and Teheran University (546-7)

(36) Asymptotic behavior of solutions of canonical systems near a closed, unstable orbit
Mr. D. L. Slotnick, International Business Machines, New York (546-13)

Session on Algebra and Theory of Numbers, and Applied Mathematics, Pupin Physical Laboratories, Room 428

(37) Local fundamental groups of Kroneckerian varieties
Professor Shreeram Abhyankar, Cornell University (546-2)

(38) A generalized ring of quotients. II
Professor G. D. Findlay and Professor Joachim Lambek, McGill University (546-31)

(39) Concerning non-invariant submodules of simple, matrix, and local matrix rings
Professor C. C. Faith, Pennsylvania State University

(40) Spinors and quaternions
Professor T. G. Room, University of Sydney (546-21)

(41) Some equalities in a unitary space leading to equalities concerning singular values of sets of matrices
Dr. A. R. Amir-Moéz, Queens College (546-4)

(42) Progressing waves in an infinite nonlinear string. Preliminary report
Professor B. A. Fleishman, Rensselaer Polytechnic Institute (546-11)

(43) On the steady-state solution of a heat conduction problem with Stephan-Boltzmann radiation losses
Professor M. S. Klamkin, Avco Research and Advance Development Division and Polytechnic Institute of Brooklyn (546-18)

(44) An existence and uniqueness theorem for a non-linear Stefan problem
Dr. W. T. Kyner, Shell Development Company, Emeryville, California (546-40)

SUPPLEMENTARY PROGRAM

(To be presented by title)

(45) A 2-manifold-with-boundary in E^3 is tame if its complement is 1-ULC
Professor R. H. Bing, University of Wisconsin and Institute for Advanced Study

(46) A theorem on factorization in semigroups that results from a theorem of Kleene's in the theory of automata
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York

(47) A machine process for maximizing a function of one real variable
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York

(48) Concrete representation of the axiom of choice within Boolean algebra
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York

(49) A fundamental theorem in the theory of continued fractions
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York

(50) Fixed points and torsion on Kahler manifolds
Professor T. T. Frankel, Stanford University

(51) On the number of dissimilar graphs between a given graph-subgraph pair
Professor Frank Harary, University of Michigan and Institute for Advanced Study

(52) On sets in both two-function-quantifier forms
Mr. D. L. Kreider, Massachusetts Institute of Technology

(53) A proof of existence and uniqueness for the solution of a discrete non-linear boundary value problem
Dr. G. H. Pimbley, Jr., Los Alamos Scientific Laboratory

(54) Solution of the initial value problem for the linearized multi-velocity transport equation with a slab geometry
Dr. G. H. Pimbley, Jr., Los Alamos Scientific Laboratory

(55) A power series with small partial sums
Dr. H. S. Shapiro, New York University

(56) Approximation by rational functions
Dr. H. S. Shapiro, New York University

(57) On least square circles and spheres
Mr. Larry Shepp, Polytechnic Institute of Brooklyn, and Professor M. S. Klamkin, Avco Research and Advance Development Division and Polytechnic Institute of Brooklyn

(58) Relation between recurrent events and an inductive law
Professor L. W. Small, Yonkers, New York

Program Committee for the Symposium:

Richard Bellman
George E. Kimball
S. A. Schelkunoff
Claude Shannon
C. B. Tompkins
Marshall Hall, Jr., Chairman

Richard D. Schafer
Associate Secretary

Storrs, Connecticut
March 5, 1958

PRELIMINARY ANNOUNCEMENT OF MEETING

FIVE HUNDRED FORTY-SEVENTH MEETING

Corvallis, Oregon
June 21, 1958

The five hundred forty-seventh meeting of the American Mathematical Society will be held on Saturday, June 21, 1958, at Oregon State College in Corvallis, Oregon. There will be a meeting of the Mathematical Association of America on Friday, June 20, and a meeting of the Society for Industrial and Applied Mathematics on Friday or Saturday.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, the Society will be addressed on Saturday morning by Professor Leo Sario of the University of California, Los Angeles. His topic is "Problems on Riemann surfaces". Sessions for contributed papers will be held on Saturday afternoon.

Dormitory accommodations will be available at $2.00 per person per night, beginning Thursday evening, June 19, at Sackett Hall C. Food service will be provided at the Sackett Hall cafeteria from breakfast on June 20 to dinner on June 21. There will be an MAA dinner Friday evening, June 20, at the Memorial Union Tearoom, costing $2.00. Reservations are to be sent to Professor R. D. Stalley, Department of Mathematics, Oregon State College, Corvallis, Oregon, by June 13, stating time of arrival, dormitory accommodations required, and number of reservations for the dinner Friday evening.

For those desiring other accommodations, the following are listed. Reservations should be made directly to the hotel or motel.

	Single	Double
Benton Hotel	$4.00-$5.00	$5.50-$6.50
City Center Motel	5.00	7.00
Con Mar Motel	5.00	7.00

Corvallis is about 80 miles south of Portland on US Highway 99W. It is served by West Coast Airlines and by Greyhound Bus. The Southern Pacific railroad line passes through Albany, ten miles east of Corvallis, and Greyhound service is available from Albany to Corvallis. (However, connections between Southern Pacific and Greyhound are less than ideal.)

V. L. Klee, Jr.
Associate Secretary

Seattle, Washington
March 5, 1958

ACTIVITIES OF OTHER ASSOCIATIONS

THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE has announced the following places and dates for its annual meetings: December 26-31, 1958, Washington, D. C.; December 26-31, 1959, Denver, Colorado; December 26-31, 1960, Philadelphia, Pennsylvania; December 26-31, 1961, Milwaukee, Wisconsin.

THE INSTITUTE OF RADIO ENGINEERS invites members of the American Mathematical Society to affiliate with the Professional Group on Electronic Computers of the IRE. The affiliation is provided by the IRE as a service to members of professional societies in fields allied to electronic computers but whose primary interest may not be in electronics. Application forms may be obtained from the Institute of Radio Engineers, One East 79th Street, New York 21, New York. One should write to the attention of Mr. L. G. Cumming, Technical Secretary.

The Professional Group on Electronic Computers is an association of IRE members and affiliates with professional interest in the fields of analog and digital electronic computers. The group is concerned with the advancement of the electronic computer field and serves to aid in promoting close cooperation and exchange of technical information among its members.

The fee is $6.50 per year and includes the quarterly Transactions of this professional group. The rate is considerably below regular membership rate in the IRE, thereby giving you, as a member of the AMS, an opportunity to read this major technical publication in the computer field published by the IRE without the expense of full membership in a second professional society.

THE THIRTEENTH ANNUAL MEETING OF THE ASSOCIATION FOR COMPUTING MACHINERY will be held at the University of Illinois, Urbana, Illinois, on June 11-13. Contributed papers are invited. It has been the policy of the Association to allow 15 minutes for each presentation and thereafter 5 minutes for discussion. Each author desiring to submit contributed papers should send four copies of an abstract of not more than 200 words, and of a three page summary, not including proofs, of the principal results and their applications to Professor Jim Douglas, Jr., Department of Mathematics, Rice Institute, Houston, Texas, by March 3, 1958.

Editorial Note: This announcement was received in the offices of the American Mathematical Society on January 27, 1958, two weeks after the deadline for the February NOTICES. The earliest issue it could appear in is therefore the present one. The Editor regrets that the announcement will appear too late for compliance with the deadline for contributed papers indicated above in the text.

CANADIAN MATHEMATICAL BULLETIN. This Bulletin will begin publication with volume one in 1958. It is to be the House Organ of the Canadian Mathematical Congress. Some space will be given to short research papers, and to expository articles. It is planned to publish one volume per year. In the first year there will be two or three sections. The Bulletin will be distributed free to members of the Canadian Mathematical Congress. (The membership fee of the Congress is $3.) It will be available to Libraries or to individuals at $3 per volume. Orders should be sent to the Canadian Mathematical Bulletin, Department of Mathematics, McGill University, Montreal 2, Quebec. Cheques should be made payable to the Canadian Mathematical Bulletin.

TRANSLATION OF THE "PRIKLADNAYA MATEMATIKA I MEKHANIKA". This Journal title becomes in English the Journal of Applied Mathematics and Mechanics. The Journal is published bimonthly in the U.S.S.R. The American Society of Mechanical Engineers has announced that under a grant from the National Science Foundation the entire journal will be translated on a regular basis. Publication will begin with the first 1958 issue, of which 2500 copies will be printed. Professor George Herrmann of Columbia University will serve as the editor of the translated magazine. The Journal contains some of the latest theoretical and practical advances made by Russian scientists in mathematics, fluid dynamics and solid state physics. Copies will be sold on a subscription basis to any interested persons or groups at an annual rate of $35 for six issues. Members of the American Society of Mechanical Engineers are entitled to a 20% discount. Subscriptions may be ordered from the Order Department, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, New York.

MATHEMATICS DIVISION OF THE NATIONAL ACADEMY OF SCIENCES - NATIONAL RESEARCH COUNCIL. It has been announced that Professor Samuel S. Wilks, of Princeton University, has been appointed Chairman of the Division to succeed Professor Paul A. Smith, of Columbia University, who has served as Division chairman since 1955.

NEWS ITEMS AND ANNOUNCEMENTS

A SYMPOSIUM ON NUMERICAL APPROXIMATION, sponsored by the Mathematics Research Center of the U. S. Army, will be held on April 21-23, 1958, at the University of Wisconsin, Madison. The aim of the Symposium is to survey recent work in these fields and to present original contributions.

A number of rooms in the Short Course Dormitory of the University of Wisconsin will be available to participants from April 20-23. These rooms are designed for occupancy by two or three persons. The rate is $2.50 per person per night for two persons per room.

The program committee requests that persons interested in the Symposium should write at once to Professor R. E. Langer, the Director of the Mathematics Research Center, at 1118 West Johnson Street, Madison 6, Wisconsin. Preference for dormitory or hotel accommodations should be mentioned in the letter. Further announcements and a final program will be mailed to all those who indicate their interest.

The tentative program as of January 20, 1958, is as follows:

Monday, April 21: Session on Linear Approximation

A. M. Ostrowski--Introduction: On trends and problems in linear approximation	45 min.
I. J. Schoenberg--Smoothing	25 min.
Z. Kopal--Numerical integration and differentiation by means of rational functions	25 min.
Ph. Davis--Remainder formulae	25 min.
H. E. Salzer--Multidimensional interpolation	25 min.
M. Golomb and H. F. Weinberger--Generalized linear interpolation and error bounds	25 min.
A. Sard--Summary: The rationale of approximation	45 min.

Tuesday, April 22: Session on Extremal Approximations

J. L. Walsh--Introductory survey	45 min.
E. L. Stiefel--Automatic Tschebycheff approximation	25 min.
T. S. Motzkin--Existence of essentially non-linear families for oscillatory approximation	25 min.
M. Golomb--Approximation by functions of fewer variables	25 min.
L. Fox--Minimax methods in table construction	25 min.
J. C. P. Miller-- Summary and conclusions	45 min.

Wednesday, April 23: Session on Algorithms and Other Methods

R. C. Buck--Survey of recent Russian literature on approximation	45 min.
J. B. Rosser--Some sufficient conditions for the existence of an asymptotic formula or an asymptotic expansion	25 min.
P. Hammer--Numerical evaluation of multiple integrals	25 min.
L. Collatz--Approximation in partial differential equations	25 min.
J. Todd--Summary and conclusions	45 min.

(A few other lectures will be announced.)

ACKNOWLEDGMENTS TO REFEREES. The publication of scientific journals would be impossible without the dedicated and unselfish efforts of referees. The job of a referee is a thankless one. He must necessarily remain anonymous, while others participating in various stages of publication can at least receive credit for their contribution. To read manuscript from a referee's viewpoint is often unpleasant and always time consuming. The combined good judgment of the referees is essential to making a journal successful.*

The Council of the Society has authorized the publication of combined lists of referees once a year in the NOTICES. The publication of these lists provides the referees with a measure of the recognition but it at the same time protects their anonymity.

The following two lists are based on information available at the Headquarters Offices of the Society as of March 5, 1958.

Referees of the BULLETIN, the PROCEEDINGS, and the TRANSACTIONS of the American Mathematical Society:

R. P. Agnew, Y. Akizuki, A. A. Albert, W. Ambrose, R. D. Anderson, N. C. Ankeny, Louis Auslander, Maurice Auslander, W. Bailey, Iacoppi Barsotti, P. T. Bateman, R. A. Beaumont, R. E. Bellman, R. H. Bing, Garrett Birkhoff, R. L. Blair, H. D. Block, A. Borel, Richard Brauer, W. E. Briggs, R. C. Buck, C. E. Burgess, Eugenio Calabi, A. P. Calderon, H. H. Campaigne, Leonard Carlitz, L. Cesari, Harvey Cohn, W. J. Coles, P. Conner, P. F. Conrad, H. S. M. Coxeter, C. W. Curtis, M. L. Curtis, D. A. Darling, P. Davis, M. M. Day, J. C. E. Dekker, W. E. Deskins, A. Devinatz, D. J. Dickinson, S. Diliberto, Jim Douglas, Jr., A. Douglis, M. P. Drazin, R. J. Duffin, J. Dugundji, M. Dwass, E. Dyer, F. J. Dyson, A. Edrei, J. Eells,

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*The editor is indebted to Dr. Franz L. Alt, Chairman of the Editorial Board of the Journal of the Association for Computing Machinery, for suggesting this introduction.

Ky Fan, N. J. Fine, I. B. Fleischer, E. E. Floyd, M. K. Fort, A. L. Foster, E. Frank, D. Gale, P. R. Garabedian, W. Gautschi, L. Gillman, Wallace Givens, A. W. Goodman, O. Gross, V. K. Gugenheim, W. Gustin, Felix Haas, D. T. Haimo, Franklin Haimo, Marshall Hall, M. E. Hamstrom, Bruno Harris, N. S. Hawley, G. A. Hedlund, A. E. Heins, M. H. Heins, A. Heller, M. Henriksen, I. N. Herstein, C. S. Herz, M. R. Hestenes, Graham Higman, I. I. Hirschman, U. Hochstrasser, Alfred Horn, A. S. Householder, S. T. Hu, J. A. Hummel, J. R. Isbell, Arno Jaeger, J. A. Jenkins, M. Jerison, Fritz John, Bjarni Jonsson, W. Kaplan, S. Karlin, J. L. Kelley, Erwin Kleinfeld, L. A. Kokoris, E. R. Kolchin, Georg Kreisel, H. Kuhn, N. H. Kuiper, Joseph Landin, S. Lang, J. P. La Salle, G. Latta, P. D. Lax, D. H. Lehmer, Emma Lehmer, Joseph Lehner, W. Leighton, W. J. LeVeque, J. J. Levin, Norman Levinson, C. C. Lin, W. G. Lister, O. I. Litoff, L. H. Loomis, R. C. Lyndon, C. C. MacDuffee, W. MacKenzie, J. S. MacNerney, M. L. MacQueen, J. E. McLaughlin, G. Mackey, Wilhelm Magnus, Kurt Mahler, Morris Marden, D. Miller, E. E. Moise, J. C. Moore, P. S. Mostert, G. D. Mostow, T. S. Motzkin, D. C. Murdoch, J. I. Nash, Zeev Nehari, C. J. Nesbitt, B. H. Neumann, L. Nirenberg, F. Oberhettinger, R. H. Oehmke, O. T. O'Meara, D. S. Ornstein, J. L. Paige, R. Palais, Gordon Pall, R. S. Phillips, George Piranian, H. Pollard, E. C. Posner, G. B. Preston, E. D. Rainville, M. O. Reade, W. T. Reid, W. Rindler, C. A. Rogers, Alex Rosenberg, M. L. Rosenlicht, E. Rothe, L. A. Rubel, M. E. Rudin, H. J. Ryser, G. Sabidussi, Arthur Sard, R. D. Schafer, E. V. Schenkman, P. Scherk, I. J. Schoenberg, L. Schoenfeld, A. Schwartz, R. Schwartz, Dana Scott, W. R. Scott, W. T. Scott, A. Seidenberg, G. B. Seligman, A. Shapiro, E. Snapper, E. H. Spanier, F. L. Spitzer, G. Springer, E. Stein, S. Sternberg, R. R. Stoll, W. Strodt, A. H. Taub, O. Taussky, R. Thom, E. Thomas, John Thompson, W. J. Trijitzinsky, W. T. Tutte, W. R. Utz, A. D. Wallace, George Whaples, M. C. Weiss, J. G. Wendel, G. W. Whitehead, A. L. Whiteman, P. M. Whitman, D. V. Widder, A. B. Willcox, R. F. Williams, A. Wintner, A. C. Woods, F. B. Wright, Bertram Yood, G. S. Young, O. Zariski, H. J. Zassenhaus, Daniel Zelinsky, J. L. Zemmer, Antoni Zygmund.

Referees of the *Canadian Journal of Mathematics*, the *Duke Mathematical Journal*, the *Michigan Mathematical Journal*, *Mathematical Tables and other Aids to Computation*, and the *Journal of the Association for Computing Machinery*:

M. Abramowitz, R. P. Agnew, L. V. Ahlfors, A. D. Alexandrox, C. B. Allendoerfer, F. L. Alt, R. Arens, L. Auslander, R. Baer, F. Bagemihl, W. N. Bailey, B. Banaschewski, V. Bargman, R. W. Bass, R. H. Battin, E. Begle, S. D. Bernardi, G. Birkhoff, C. G. Blanyer, R. P. Boas, Jr., A. Borel, R. Bott, L. R. Bragg, A. Brauer, G. Brauer, F. Browder, G. W. Brown, W. P. Brown, F. H. Brownell, R. H. Bruck,

F. Bureau, A. W. Burks, H. Busemann, A. P. Calderon, S. G. Campbell, L. Carlitz, S. Chandrasekhar, S. S. Chern, K. L. Chung, R. F. Clippinger, E. A. Coddington, H. Cohen, H. Cohn, A. J. Coleman, P. Conner, S. D. Conte, W. J. R. Crosby, C. W. Curtis, M. L. Curtis, E. Cuthill, H. Davenport, M. Davis, P. J. Davis, R. Davis, M. M. Day, F. Delaney, J. B. Diaz, J. A. Dieudonné, R. P. Dilworth, B. Dimsdale, J. L. Doob, F. G.Dressel, G. F. D. Duff, W. F. Eberlein, W. L. Edge, H. P. Edmundson, A. Edrei, D. A. Edwards, H. G. Eggleston, J. Elliott, P. Erdös, K. Fan, H. Feshbach, D. A. Flanders, A. Fletcher, E. E. Floyd, G. E. Forsythe, M. K. Fort, Jr., C. Fox, R. H. Fox, S. P. Frankel, T. M. Gallie, Jr., W. Gautschi, F. Gehring, J. J. Gergen, E. N. Gilbert, L. Gillman, A. Gleason, E. T. Goodwin, S. Gorn, J. Griffin, E. Grosswald, R. C. Gunning, P. Hall, P. R. Halmos, P. C. Hammer, R. W. Hamming, E. M. Hartley, P. Hartman, G. A. Hedlund, P. Henrici, M. Henriksen, I. N. Herstein, C. Herz, E. Hewitt, G. Heyliger, D. G. Higman, T. L. Hildebrandt, I. I. Hirschman, U. Hochstrasser, A. J. Hoffman, F. Hollander, G. M. Hopper, A. S. Householder, S. T. Hu, R. Isaacs, N. Jacobson, R. D. James, J. P. Jans, R. L. Jeffery, B. W. Jones, M. Kac, R. V. Kadison, S. Kakutani, W. Kaplan, I. Kaplansky, W. Karush, K. E. Kavanagh, E. S. Keeping, M. A. Kervaire, J. L. Kelley, L. M. Kelly, V. L. Klee, Jr., H. L. Krall, E. Kreyszig, C. C. Krieger, D. Kurepa, R. E. Langer, R. S. Lehman, E. Lehmer, D. H. Lehmer, C. E. Lemke, N. Levinson, L. H. Loomis, G. G. Lorentz, W. A. J. Luxemburg, R. Lyndon, G. W. Mackey, S. MacLane, M. J. MacPhail, M. Mannon, W. H. Marlow, N. M. Martin, T. E. Martin, W. S. Massey, N. S. Mendelsohn, B. Meyer, N. McCoy, J. E.McLaughlin, R. M. McLeod, R. McNaughton, G. W. Medlin, F. Meek, C. H. Meng, E. A. Michael, A. N. Milgram, J. C. P. Miller, D. Montgomery, E. F. Moore, L. Moser, W. Moser, J. Moshman, T. S. Motzkin, D. Muller, D. C. Murdoch, F. J. Murray, J. Nash, M. Newman, A. Nijenhuis, L. Nirenberg, I. Niven, R. Z. Norman, O. Ore, E. E. Osborne, G. Pall, W. V. Parker, A. H. Payne, C. E. Pearson, A. G. Peck, A. J. Perlis, S. Perlis, C. L. Perry, R. S. Phillips, R. S. Pinkham, E. Pinney, H. Pollard, R. Prosser, M. H. Protter, H. Rademacher, E. D. Rainville, B. A. Rattray, E. Reich, J. H. Roberts, A. Robinson, G. de B. Robinson, C. A. Rogers, P. G. Rooney, A. Rozenberg, M. Rubinoff, W. Rudin, H. J. Ryser, R. G. Sachs, H. E. Salzer, H. Samelson, O. F. G. Schilling, I. E. Segal, W. W. Seifert, J. L. Selfridge, R. G. Selfridge, I. M. Sheffer, J. R. Shoenfield, M. F. Smiley, J. W. Smith, T. H. Southard, D. C. Spencer, R. Stanton, C. W. Steeg, N. E. Steenrod, R. Steinberg, R. L. Sternberg, B. M. Stewart, P. F. Strong, D. J. Struik, P. B. Sumner, O. Taussky, D. Teichroew, H. C. Thatcher, J. M. Thomas, R. M. Thrall, H. Tietz, J. Todd, W. Tutte, B. van der Pol, D. D. Wall, A. D. Wallace, C. Walmsley, M. Ward, S. L. Warner, S. E. Warschawski, W. Wasow, H. F. Weinberger, L. Weisner, J. Wendel,

J. Wermer, G. W. Whitehead, A. L. Whiteman, L. L. Whyte, D. V. Widder, H. Wielandt, J. H. Wilkinson, M. Wing, F. B. Wright, Jr., M. Wyman, D. M. Young, Jr., L. C. Young, H. Zassenhaus, D. Zelinsky, K. Zeller, L. Zippin, A. Zygmund.

A FOREIGN TECHNICAL INFORMATION CENTER IN THE U. S. DEPARTMENT OF COMMERCE. The Department of Commerce announced that it has recently requested a special appropriation of $300,000 to set up such a Center. In addition, the President's budget for 1959 included $1,250,000 for the Department's Foreign Technical Information program. The new Center is intended to act as a central clearinghouse for the collection, the evaluation, and the distribution of foreign and scientific and technical literature for the use of American scientists and engineers. Plans have been worked out for the public distribution of information from such organizations as the National Science Foundation, the Atomic Energy Commission, the armed services and the intelligence agencies. Of particular interest to mathematicians is the fact that arrangements have been made to obtain from these and other agencies copies of abstracts and translations of foreign technical articles, monographs and books which have been prepared by the agencies. It is estimated that these will pass through the Center at an annual rate of 50,000 abstracts and 10,000 complete translations.

The news release which announced the plans for the Center also made the following statement: "The new Center will operate a coordination service to eliminate duplication of translating among U. S. public and private agencies and by friendly foreign governments."

THE COMMITTEE FOR LIAISON WITH HIGH SCHOOLS OF THE DEPARTMENT OF MATHEMATICS OF THE CARNEGIE INSTITUTE OF TECHNOLOGY. This Committee was formed in September of 1955 to establish contacts with high school mathematics teachers and administrators in the interests of methods for improving the quality and quantity of mathematics teaching. Through a series of luncheon meetings with teachers in the Pittsburgh area the Committee has met with persons representing eighty-five per cent of the area's high school students. The luncheons have brought requests for additional meetings and for speaking engagements on the part of the members of the Committee. The Committee has been approached by school administrators to prepare recommendations on teaching problems such as the addition of analytic geometry to the high school mathematics curriculum. Currently the membership of the Committee consists of Professor John H. Neelley, Professor Richard A. Moore, Professor Joseph Auslander, Professor Marlow C. Sholander, and Professor Allen F. Strehler, Chairman.

NEW CORPORATE MEMBERS OF THE AMERICAN MATHEMATICAL SOCIETY. The Society is happy to announce that the following business corporations have accepted invitations to become corporate members of the Society: the Bell Telephone Laboratories, the Ford Motor Company, the General Motors Corporation, and the Hughes Aircraft Company.

VISITING FOREIGN MATHEMATICIANS. In the December, 1957, issue of the NOTICES, there appeared the annual list of visiting foreign mathematicians prepared by the Division of Mathematics of the National Academy of Sciences - National Research Council. As an addition to the list, it should be stated that Dr. Karl Zeller of Germany was a Visiting Associate Professor at Wayne State University from January, 1957, through February, 1958, and left for Germany during the first week of March, 1958.

CORRECTIONS. In the February 1958 issue of the NOTICES, there were two imperfections in the printing of the abstract of Professor John Todd, on page 72 (abstract no. 539-43). Only one of these is important. It concealed the date on which the abstract was received. This date was October 2, 1957.

On page 10 of the February 1958 issue of the NOTICES, it is stated that the dates of the Third U. S. National Congress of Applied Mechanics will be June 1-14, 1958. The dates should be June 11-14.

PERSONAL ITEMS

(This section is restricted to members of the Society)

Professor A. A. Albert of the University of Chicago has been elected chairman of the mathematics section of the National Academy of Sciences for a three-year term.

Assistant Professor L. H. Lange of Valparaiso University is on leave and has been awarded a Danforth Foundation Teacher Study Grant at the University of Notre Dame.

The National Science Foundation has announced grants in science to the following institutions, to support studies by the professors indicated: University of Buffalo, Harriet F. Montague; Carleton College, K. O. May; Case Institute of Technology, P. E. Guenther; University of Chicago, A. A. Albert, Saunders MacLane, E. P. Northrop, A. L. Putnam; University of Colorado, W. E. Briggs; Dartmouth College, J. G. Kemeny; University of Illinois, Joseph Landin; University of Kansas, G. B. Price; University of Minnesota, S. E. Warschawski; New York University, Richard Courant; Northwestern University, Alex Rosenberg, Daniel Zelinsky; University of Notre Dame, A. E. Ross; Rensselaer Polytechnic Institute, E. B. Allen; Rutgers University, E. P. Starke; University of Vermont and State Agricultural College, N. J. Schoonmaker; University of Wyoming, W. N. Smith.

Dr. E. S. Andersen of the Institut of Copenhagen has been appointed to a professorship at Aarhus Universitet, Aarhus, Denmark. He is on leave and has been appointed to a visiting professorship at the University of Minnesota.

Assistant Professor D. G. Austin of Ohio State University.has been appointed to an associate professorship at the University of Miami.

Miss Ruth E. Biggers of Tulane University has been appointed to an assistant professorship at Hiwassee College.

Dr. Mary L. Boas of Evanston, Illinois has been appointed to an assistant professorship at De Paul University.

Professor M. G. Boyce of Vanderbilt University is on leave and is a visiting fellow at Princeton University.

Mr. Roger L. Boyell of Ramo-Wooldridge Corporation has accepted a position as engineer for Sperry Gyroscope Company, Great Neck, New York.

Mr. A. J. Brinson of Seton Hall University has accepted a position as mathematics consultant in the Sales Promotion Department of Monroe Calculating Company, Orange, New Jersey.

Dr. L. C. Butler of Alfred, New York has been appointed to an assistant professorship at the College of Ceramics, State University of New York.

Assistant Professor C. E. Capel of the University of Miami has accepted a position in the Mathematics Department, Research Laboratories, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.

Dr. R. C. Carson of Lehigh University has been appointed to an assistant professorship at Western Reserve University.

Major B. B. Clark of the United States Air Force has accepted a position as a member of the technical staff at the Ramo-Wooldridge Corporation, Los Angeles, California.

Dr. G. F. Cramer of Sperry-Rand Corporation has accepted a position as mathematician at International Business Machines, Ossining, New York.

Dr. R. H. Davis of Bell Telephone Laboratories has accepted a position as operations analyst for Stanford Research Institute, Menlo Park, California.

Dr. R. F. Dennemeyer of Lockheed Aircraft, Inc. has been appointed to an associate professorship at Long Beach State College.

Mr. R. A. Dibrell, Jr. of the U. S. Army Air Defense Command has accepted a position as a member of the technical staff of Hughes Aircraft Company, Fullerton, California.

Dr. Jim Douglas, Jr. of Humble Oil and Refining Company has been appointed to an assistant professorship at Rice Institute.

Dr. Worthie Doyle of Hughes Aircraft Company has accepted a position on the staff of the Lincoln Laboratory, Massachusetts Institute of Technology.

Dr. R. F. Drenick of RCA Victor Division has accepted a position as a member of the technical staff of Bell Telephone Laboratories, New York, New York.

Assistant Professor Olive J. Dunn of Iowa State College of Agriculture and Mechanic Arts has been appointed to an assistant professorship at the University of California, Los Angeles.

Dr. R. E. Fagen of the Bell Telephone Laboratories has accepted a position as a member of the technical staff at Hughes Aircraft Company, Culver City, California.

Mr. J. A. Ferling of the University of Southern California has been appointed to an assistant professorship at Claremont Men's College.

Mr. M. L. Freimer of Harvard University has accepted a position on the staff of the Lincoln Laboratory, Massachusetts Institute of Technology.

Mr. I. M. Garfunkel of the Rheem Manufacturing Company has accepted a position as senior mathematician in the Evaluation Group of the System Development Corporation, Santa Monica, California.

Professor Abe Gelbart of Syracuse University has been appointed to a professorship at Yeshiva University.

Professor O. G. Harrold of the University of Tennessee is on leave and has been appointed a Fellow at the Institute for Advanced Study, Princeton, New Jersey.

Professor H. H. Hartzler of Goshen College has been appointed to an associate professorship at Mankato State College.

Assistant Professor Emilie V. Haynsworth of Wilson College has accepted a position as mathematician for the National Bureau of Standards, Washington, D. C.

Dr. M. A. Hyman of Remington Rand Univac has accepted a position as senior mathematician in the Research Center of the International Business Machines Corporation, Ossining, New York.

Mr. B. B. James of North American Aviation has accepted a position as senior research engineer for Autonetics, Bellflower, California.

Mr. H. M. Kamowitz of Newton Centre, Massachusetts has accepted a position as associate scientist for Avco Manufacturing Company, Lawrence, Massachusetts.

Mrs. Maria L. M. Leitre Lopes of the California Institute of Technology has been appointed to an assistant professorship at the Universidade Brazil.

Mr. B. W. Levinger of Tung-Sol Electric, Inc., has accepted a position as senior engineer for Sylvania Electric Products, Inc., Bayside, New York.

Dr. R. J. Levit of the International Business Machines Corporation has been appointed to an assistant professorship at the San Francisco State College.

Dr. J. C. Lillo of Princeton University has been appointed to an assistant professorship at the University of Kansas.

Assistant Professor R. W. MacDowell of the University of Rochester has been appointed to an associate professorship at Antioch College.

Dr. M. E. Mahowald of General Electric Company has been appointed to an assistant professorship at Xavier University.

Mr. T. W. Mullikin of Harvard University has accepted a position as associate mathematician at the RAND Corporation, Santa Monica, California.

Dr. Peter Musen of Continental Oil Company has been appointed to an assistant professorship at the University of Cincinnati.

Mr. J. O. Neilson of RAND Corporation has accepted a position as assistant mathematician for the System Development Corporation, Lexington, Massachusetts.

Mr. E. P. Neuburg of the Defense Department has accepted a position as mathematician with the National Security Agency, Fort Meade, Maryland.

Professor T. K. Pan of the University of Oklahoma has been appointed to a professorship at the National Taiwan University, Taiwan, Formosa, China.

Mr. R. W. Paul, Jr. of the University of California, Livermore, has accepted a position as associate engineering mathematician for

for the Marquardt Aircraft Company, Van Nuys, California.

Miss Mabel A. Pauley of the University of Miami has accepted a position as mathematician for the National Security Agency, Washington, D. C.

Dr. G. O. Peters of Westinghouse Electric Corporation has accepted a position as engineer for RCA Service Company at Patrick Air Force Base, Cocoa Beach, Florida.

Dr. L. L. Philipson of Hughes Research and Development Laboratories has accepted a position as research scientist for Litton Industries, Inc., Beverly Hills, California.

Dr. M. O. Rabin of Princeton University has been appointed a member of the Institute for Advanced Study.

Mr. R. R. Reynolds of Corvallis, Oregon has been appointed to an assistant professorship at Oregon State College.

Dr. Azriel Rosenfeld of Ford Instrument Company has been appointed to a visiting assistant professorship at Yeshiva University.

Dr. W. R. Seugling of Allstates Engineering Company has accepted a position as senior research engineer for Rocketdyne, North American Aviation, Inc., Canoga Park, California.

Mr. Robert Simon of Remington Rand Corporation has accepted a position as supervisor of the computing staff, Engine Division, Fairchild Engine and Airplane Corporation, Deer Park, New York.

Mr. M. H. Slud of Cornell Aeronautical Laboratory has accepted a position as mathematical analyst for General Electric Company, Philadelphia, Pennsylvania.

Dr. Donald Solitar of New York University has accepted a position as a member of the technical staff of G. C. Dewey and Company, New York, New York.

Dr. J. R. Stock of the Union Carbide and Carbon Corporation has accepted a position as engineer for Stock Equipment Company, Cleveland, Ohio.

Mr. R. E. Thomas of North American Aviation, Inc.has accepted a position as assistant division consultant for Battelle Memorial Institute, Columbus, Ohio.

Dr. Paul Weiss of General Electric Company has accepted a position as senior staff member for Avco Research Laboratory, Everett, Massachusetts.

Dr. F. J. Weyl, formerly Director of the Mathematical Sciences Division, Office of Naval Research, has been appointed Director, Naval Analysis Group, Office of Naval Research, Washington, D. C.

Dr. Philip Wolfe of Princeton University has accepted a position as mathematician for the RAND Corporation, Santa Monica, California.

Professor Roscoe Woods of the State University of Iowa has retired with the title Professor Emeritus.

Mr. J. W. Young, Jr. of the National Security Agency has accepted a position as chief, Advanced Systems Research Section, National

Cash Register Company, Hawthorne, California.

Professor E. H. Zarantonello of the University of Cuyo is on leave and has been appointed to a visiting professorship at the University of Wisconsin.

The following promotions are announced:

Paul Axt, Lehigh University, to an assistant professorship.

Dorothy L. Bernstein, University of Rochester, to a professorship.

M. L. Coffman, Abilene Christian College, to an associate professorship.

M. A. Eliopoulos, Essex College, Assumption University of Windsor, to an assistant professorship.

G. D. Findlay, McGill University, to an assistant professorship.

A. B. Finkelstein, Pratt Institute, to an associate professorship.

T. T. Frankel, Stanford University, to an assistant professorship.

B. T. Goldbeck, Jr., Texas Christian University, to an associate professorship.

Simon Green, University of Tulsa, to an associate professorship.

G. L. Krabbe, Purdue University, to an assistant professorship.

A. S. Littell, Western Reserve University, to an assistant professorship.

A. P. Mattuck, Massachusetts Institute of Technology, to an assistant professorship.

J. P. Murray, Fairfield University, to an associate professorship.

E. V. Schenkman, Louisiana State University, to a professorship.

J. D. Swift, University of California, Los Angeles, to an associate professorship.

C. J. Titus, University of Michigan, to an assistant professorship.

J. H. Weiner, Columbia University, to an associate professorship.

The following appointments to instructorships are announced:

University of California, Los Angeles: Dr. J. W. Blattner; University of California, Riverside: Dr. W. D. James; University of Cincinnati: Mr. Roger Chalkley; Lake Forest College: Mr. A. J. Hechenbach; University of Maryland: Dr. Ellen Correl; Michigan State University: Dr. J. L. Bailey; University of Michigan: Mr. J. E. Keisler, Mr. R. R. Korfhage; Ohio State University: Mr. Shen Lin; Pennsylvania State University: Mr. W. A. Beyer; Wayne State University: Mr. C. D. LaBudde; University of Wisconsin, Milwaukee: Dr. G. R. Lehner; Yale University: Dr. Leonard Gross.

Deaths:

Mr. W. M. Bullitt of Bullitt, Dawson and Tarrant, Louisville, Kentucky, died October 3, 1957 at the age of eighty-four years. He had been a member of the Society for thirty-two years.

Mr. Louis Burgess of New York, New York died May 15, 1957 at the age of fifty-six years. He had been a member of the Society for nineteen years.

Mr. R. W. Byerly of New York, New York died October 7, 1957 at the age of sixty-eight years. He had been a member of the Society for fifteen years.

Professor Arthur Hendler of Rensselaer Polytechnic Institute died June 11, 1957 at the age of thirty-six years.

Professor Emeritus W. A. Hurwitz of Cornell University died January 6, 1958 at the age of seventy-one years. He had been a member of the Society for thirty-one years.

Professor Emeritus C. B. Upton of Teachers College died September 25, 1957 at the age of eighty years. He had been a member of the Society for fifty-four years.

NEW PUBLICATIONS

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Adirovič, È. I. Nekotorye voprosy teorii lyuminescencii kristallov. 2d ed. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 350 pp. 10.45 rubles.

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Angot, A. Compléments de mathématique à l'usage des ingénieurs de l'électrotechnique et des télécommunications. 3d ed. Paris, Editions de la Revue d'Optique, 1957. 8 + 836 pp. 4800 fr.

Applied probability. (Proceedings of Symposia in Applied Mathematics, vol. 7.) New York, McGraw-Hill, 1957. 104 pp. $5.00.

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Armstrong, A. H. See Mikhlin, S. G.

Arzeliès, H. La dynamique relativiste et ses applications. No. I. Dynamique du point lentement accéléré. Application aux phénomènes d'interaction entre charges électriques (électromagnétisme relativiste). Paris, Gauthier-Villars, 1957. 14 + 304 pp. 4000 fr.

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Atkin, R. H. Mathematics and wave mechanics. New York, Wiley, 1957. 15 + 348 pp. $6.00.

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Bianchi, L. Opere. Vol. VI. Congruenze di rette e di sfere e loro deformazioni. Ed. by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1957. 328 pp. 3000 Lire.

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No

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Brehovskih, L. M. Vólny v sloistyh sredah. Moscow, Izdat. Akad. Nauk SSSR, 1957. 502 pp. 26.10 rubles.

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Chandrasekhar, S. An introduction to the study of stellar structure. New York, Dover, 1957. 2 + 509 pp. $2.75.

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Chu, B. T. Thermodynamics of elastic and of some visco-elastic solids and non-linear thermoelasticity. (Tech. Rep. No. 1, Nonr-562 (20). Providence, Division of Engineering, Brown University, 1957. 2 + 44 pp.

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Covert, E. E. The stability of binary boundary layers. (Naval Supersonic Laboratory, Tech. Rep. 217.) Cambridge, Massachusetts Institute of Technology, 1957. 5 + 30 pp.

Coxeter, H. S. M., and Moser, W. O. J. Generators and relations for discrete groups. Berlin, Springer, 1957. 8 + 155 pp. 32.00 DM.

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W. Sadowski. Warsaw, PańMstwowe Wydawnictwo Naukowe, 1957. 158 pp. 32 zł.

Dalenius, T. Sampling in Sweden: Contributions to the methods and theories of sample survey practice. Stockholm, Almqvist and Wiksell, 1957. 8 + 247 pp. 20.00 Sw. kr.

Den Hartog, J. P. See Tietjens, O. G.

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Fisz, M. See Czechowski, T.

Fock, V. A. Diffraction, refraction, and reflection of radio waves. Bedford, Antenna Laboratory, Air Force Cambridge Research Center, 1957. 5 + 20 + 391 pp.

Fok, V. A. Raboty po kvantovoĭ teorii polya. Leningrad, Izdat. Leningrad. Univ., 1957. 159 pp. 11.40 rubles.

Frenkel, J. I. Statistische Physik. Trans. by H. Jancke. Berlin, Akademie-Verlag, 1957. 22 + 783 pp. 48.00 DM.

Gauss, C. F. Gedenkband anlässlich des 100. Todestages am 23. Februar 1955. Leipzig, Teubner, 1957. 3 + 251 pp. $4.80.

Gerlah, L. R. Nekotorye voprosy proektirovaniya mehaničeskih integro-differenciografov. Moscow, Gosudarstv. Izdat. Oboronn. Promyšl., 1957. 53 pp. 2 rubles.

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Iwiński, T. See Czechowski, T.

Jancke, H. See Frenkel, J. I.

Jenkins, F. A., and White, H. E. Fundamentals of optics. 3d ed. New York, McGraw-Hill, 1957. 7 + 637 pp. $8.50.

Kaiser, J. F. See Newton, G. C. Jr.

Kempthorne, O. An introduction to genetic statistics. New York, Wiley, 1957. 17 + 545 pp. $12.75.

Kneser, H. See Hessenberg, G.

Kunz, K. S. Numerical analysis. New York, McGraw-Hill, 1957. 15 + 381 pp. $8.00.

Kurth, R. Introduction to the mechanics of stellar systems. New York, Pergamon, 1957. 9 + 174 pp. $9.00.

Lange, O. See Czechowski, T.

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Livesley, R. K. An introduction to automatic digital computers. Cambridge University Press, 1957. 8 + 53 pp. $1.75.

Luce, R. D., and Raiffa, H. Games and decisions: introduction and critical survey. A study of the Behavioral Models Project, Bureau of Applied Social Research, Columbia University. New York, Wiley, 1957. 19 + 509 pp. $8.75.

Mandl, F. Quantum mechanics. 2d ed. New York, Academic Press, 1957. 10 + 267 pp. $6.50. lib has 1954 ed.

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Modern computing methods. [Notes on applied science, no. 16, National Physical Laboratory, Teddington, England.] London, Her Majesty's Stationery Office, 1957. 6 + 129 pp. $1.89.

Moser, W. O. J. See Coxeter, H. S. M.

Newton, G. C. Jr., Gould, L. A., and Kaiser, J. F. Analytical design of linear feedback controls. New York, Wiley, 1957. 11 + 419 pp. $12.00.

Oniašvili, O. D. Nekotorye dinamičeskie zadači teorii oboloček. Moscow, Izdat. Akad. Nauk. SSSR, 1957. 195 pp. 9.35 rubles.

Oudart, A. See Giqueaux, M.

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Peltier, J. Inventaire collectif des périodiques mathématiques. I. Conventions, notations et répertoire par mots-types, à la date de juin 1956. (Documentation mathématique, no. 36.) Paris, Secrétariat mathématique, 1957. 53 pp. (polycopiées)

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Peltier, J. Résolution numérique des équations algébriques. Paris, Gauthier-Villars, 1957. 4 + 244 pp. 2500 fr.

Pogorelow, A. W. Die Verbiegung konvexer Flächen. [Schriftenreihe des Forschungsinstituts für Mathematik, no. 5.] Berlin, Akademie-Verlag, 1957. 135 pp. 18.50 DM.

Poincaré, H. Les méthodes nouvelles de la mécanique céleste. New York, Dover, 1957. Vol. I. Solutions périodiques. Non-existence des intégrales uniformes. Solutions asymptotiques. 5 + 382 pp. $2.75. Vol. II. Méthodes de MM. Newcomb, Gyldén, Lindstedt et Bohlin. 8 + 479 pp. $2.75. Vol. III. Invariants integraux. Solutions périodiques du deuxiéme genre. Solutions doublement asymptotiques. 2 + 414 pp. $2.75.

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Raiffa, H. See Luce, R. D.

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Ricci-Curbastro, G. Opere. Vol. II. Note e memorie: teoria dell'elasticità. Ed. by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1957. 3 + 586 pp. 5000 Lire.

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Sadowski, W. see Czechowski, T.

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Turnbull, D. See Solid state physics.

The United Kingdom contribution to the International Geophysical Year 1957-58. London, The Royal Society, 1957. 72 pp., 1 plate. 10s.

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Vzorova, A. I. Tablicy dlya rešeniya uravneniya Laplasa v èllipticěskih oblastyah. Moscow, Izdat. Akad. Nauk SSSR, 1957. 257 pp. 30 rubles.

Wallace, A. H. An introduction to algebraic topology. New York, Pergamon, 1957. 7 + 198 pp. $6.50.

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Willers, F. A. Methoden der praktischen Analysis. 3d ed. Berlin, de Gruyter, 1957. 429 pp. 28.00 DM.

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Zasępa, R. See Czechowski, T.

MEMORANDUM TO MEMBERS

AN ESSAY ON REDACTION

It is believed that the following article will be of interest to editors of the American Mathematical Society and to the authors contributing papers to its journals. It is printed with permission from The Kalends, the house organ of the Waverly Press, printers of scientific books and periodicals, of Baltimore, Maryland.

"A printer is prepared to design a book from manuscript; that is, he will set the page size, the type faces and sizes, the size of the illustrations, the kind of paper, and the binding cloth. He will not and is not trained to check the textual accuracy and consistency of the ms. For this we have redaction.

"The purpose of redaction in a publisher's office is to give the printer a manuscript free from errors in spelling, punctuation and grammar, consistent in usages, and with a logical scheme of headings and sub-headings. To accomplish this requires a careful, word-for-word reading of the entire ms. with checks and cross-checks on spellings, abbreviations and references.

"Obviously, authors are not printers, and there is no reason why they should be. But with no special knowledge and no additional labor, there are certain things an author can do or not do to ease the burden of the redactor.

"One of these is never, never, NEVER type anything single spaced. It is rare indeed for a manuscript not to have hidden in it somewhere a quoted passage, a list of references, or a footnote that is single spaced. It is usually the type of material that is set in a type size smaller than the main text, and presumably the author types it single spaced to make it look smaller. It is hard to believe that such a minor thing could be so troublesome: the redactor, the operator who sets the type, and the proofreader find it hard to read, and it leaves no room for type markings or corrections. Of course it is possible to work with single spaced copy. No one is going to refuse to accept it or feel it necessary to retype. But it is annoying to work with, and not the least of the annoyance is that it would have been just as easy to type all the copy double spaced.

"Another 'annoyance,' shall we say, is the use of staples to fasten sections of a manuscript together. Paper clips, rubber bands, file folders are fine, but, please -- no staples. They must all be removed--the printer can't handle copy that is stapled together--and some other means can be found to hold pages together. So why not use that other means in the first place?

"There are other annoyances of a mechanical nature: paper that won't take ink, paper in odd sizes and half sheets, additions made on little streamers and, heaven help us, stapled to the full-size sheets, corrections made in pencil so faint that it must be gone over, handwritten corrections that take an FBI man to decipher.

"There is also a problem of redaction that really is a problem, and not just an annoyance, and that does mean more work for the author. Too many times the references to the literature in the text do not agree with the references as listed in the bibliography. 'Smith in 1955 wrote....' But in the bibliography we have 'Smyth, 1954.' Is this the same reference? If so, which spelling is correct, and which is the correct date? Or is this something else entirely? In that case, where is the reference for the original Smith? And on the very next page of ms. we have 'Smythe, 1955.' *Now* where are we? In the bibliography itself, here is a citation with no initials for the authors' first names, here is one with no volume number for the journal cited, this one has abbreviated the name of the journal to the point where it is unidentifiable, here is a book title, but who published it and where and when? With so many items obviously wrong or missing, can we trust the items that appear to be all right? Or as a reference tool, is the bibliography completely useless? Certainly someone spent time to prepare this bibliography. Isn't it worthwhile then to spend a little more time to ensure its accuracy? Is it of any use whatsoever unless it *is* accurate?

"As publishers, we realize that authors, especially in the scientific fields, have many other irons in the fire. (Frankly, if they didn't, they wouldn't be very valuable to us as authors.) Being closely allied to the printing business, we know that clean copy means clean galley proof. But clean copy does not mean merely neatly typed pages, unless this neat typing is also free from errors and inconsistencies, and carries necessary instructions for the typesetter. And so redaction can spell the difference between clean galley proof and proof with numerous errors and queries. With only a little help from authors, we can do a better job, the printer can do a better job, and authors benefit from both."

ABSTRACTS OF CONTRIBUTED PAPERS

By direction of the Council, the abstracts of papers presented at the meetings of the Society are now being printed in the NOTICES instead of in the BULLETIN. An attempt is being made to publish the abstracts of all of the papers scheduled to be presented in person at each meeting in the issue of the NOTICES which carries the program of the meeting. There has been no advance in the deadlines for abstracts, but it is hoped that with a reasonable amount of cooperation from the members in sending abstracts in early, the project will be successful. Only abstracts which meet the specifications stated on the abstract blanks will be published.

It is expected that when a steady state has been attained, the abstracts of the papers presented at a given meeting including those of the papers read by title will all be contained in two successive issues of the NOTICES, of which the first is the issue carrying the program of the meeting. During the current year, however, there is a nonrecurrent problem in the form of a rather large backlog of abstracts (about 300 of them) inherited from the last meetings of 1957 and from the Cincinnati meeting in 1958. In this connection it will be remembered that when the abstracts were published in the BULLETIN they appeared some three to five months after the corresponding meetings. Thus the last abstracts to appear in the BULLETIN were for the Summer meeting at Pennsylvania State University.

In coping with this problem, first priority will be given to timely publication of the abstracts of the current in-person papers. The other abstracts will have to wait their turn, and this will mean in some cases that some of the abstracts for a given meeting may have to be published little by little throughout a string of several issues. It is nevertheless expected that in all cases publication will take place sooner than would have been the case if the abstracts had been published in the BULLETIN.

It is necessary to use here a somewhat more mechanical editorial system of presentation than that which was formerly used in the BULLETIN. A brief description of the new system follows.

The numbering of the abstracts in the NOTICES is an arbitrary office numbering which is unrelated to the ordering of the papers in the programs. It does not distinguish between papers presented by title and papers presented in person. It does identify the meetings at which the papers will be, or were, presented. Thus the papers presented at the New York meeting in April of 1958, which is the five hundred and forty-sixth meeting of the Society, are numbered 546-1, 546-2, etc. There are cross references to these numbers in the programs. The abstracts are not arranged under subject classifications and there is no identification of the person presenting the paper in the case of joint authorship.

THE NOVEMBER MEETING IN LOS ANGELES, CALIFORNIA

November 15-16, 1957

539-53. Reni De Vogelaere: Over-relaxations.

To solve systems of linear equations $Ay + Bz = a$, $Cy + Dz = b$, where A, B, C, D are matrices and y, z, a, b are vectors, the following iteration method is proposed: $y^{(n+1)} = w_1 A^{-1} (a - Bz^{(n)}) + (1 - w_1)y^{(n)}$, $z^{(n+1)} = z_2 D^{-1}(b - Cy^{(n+1)}) + (1 - w_2)z^{(n)}$. This method converges for a proper choice of w_1, w_2 at least as fast as for David Young's special case $w_1 = w_2$ under identical hypothesis on the matrices. The method converges also for conditions on the matrices for which neither the over-relaxation method nor the method of successive or simultaneous displacements converges. It is also generalized to matrices which, when fitted on a torus, have a diagonal and a subdiagonal of p blocks of nonzero elements using parameters $w_1, w_2, \ldots, w_p$. (Received November 16, 1957.)

THE NOVEMBER MEETING IN CORAL GABLES, FLORIDA

November 29-30, 1957

540-33. I. E. Perlin: Function approximation by broken-line functions.

A broken-line function is a real continuous function on an interval $a \leq x \leq b$, which is specified at N-points $a \leq x_i \leq b$ and is defined between successive points x_i and x_{i+1} by straight line segments joining the points (x_i, y_i) and (x_{i+1}, y_{i+1}). Given a real continuous function f(x) defined on $a \leq x \leq b$, a broken-line function, Q(x), approximating f(x) within a prescribed limit of error $\epsilon > 0$, such that the number of points x_i defining Q(x) does not exceed a given integer N, and the change in the slopes of successive linear segments does not exceed a given number m is sought. In this paper a condition is derived for the existence of such broken-line functions. If f(x) is of class C''' on $a \leq x \leq b$, and there exists an $h > 0$ such that $3^{1/2}M_3(h^3/6) + 2^{-1}|\Delta_{h/2} f(x)| \leq \epsilon$, $h^{-1}|\Delta_h^2 f(x)| \leq m$, $(b - a)/h \leq N$, where $M_3 = \max f'''(x)$, and $\Delta^2 f(x)$, $a \leq x \leq b$, is the second order difference of f(x), then there exists a broken-line function Q(x) with the prescribed requirements. Q(x) is determined for sin x, $0 \leq x \leq \pi/2$ and e^{-x}, $0 \leq x \leq 1$. (Received October 17, 1957.)

540-34. M. L. Glasser: A note on integral equations and transforms. Preliminary report.

It is noted that certain classes of integral equations may be formally inverted with the aid of appropriate changes of variable. In each class studied the variable change, in general suggested by the form of the kernel, introduces the kernel of a well known integral transform thus allowing the equation to be formally inverted through an integration. The case $F(x) = \int g(y)K(xy)dy$ is discussed. (Received November 25, 1957.)

540-35. Heinz Renggli: Note on extension of quasiconformal mappings.

Let f be a differentiable K-quasiconformal mapping of a region D, situated in the Riemann sphere R, onto another region D', $D' \subset R$ [L. Bers, Trans. Amer. Math. Soc. vol. 84 (1957) p. 78]. By g we denote a K-quasiconformal mapping of D onto D' in the sense given by L. Ahlfors [J. Analyse Math. vol. 3 (1953-1954) p. 1]. Then every f is also a g. Now, let the point-set N, $N \subset R$, be compact and let D, $D = R - N$, be connected. When can a mapping f of D be extended to some mapping of R onto R? The answer depends on what class of mappings we consider. For a given N we have: (i) any function f, defined in D, can be extended to a topological mapping of R onto R, if and only if N is a null-set $N(SB)$ [L. Ahlfors and A. Beurling, Acta Math. vol. 83 (1950) p. 101]; (ii) any function f, defined in D, can be extended to a function g, mapping R onto R, if and only if N is a null-set $N(D)$ [L. Ahlfors and A. Beurling, l.c.]. For the proof of (ii) we make use of the theorem, proved by L. Bers [l.c.], that a mapping g transforms any set of two-dimensional measure zero onto a zero-dimensional set. (Received November 26, 1957.)

THE NOVEMBER MEETING IN COLUMBIA, MISSOURI

November 29-30, 1957

541-14. R. E. Lane: Continued fraction expansions for functions.

Continued fraction expansions for a variety of functions are derived by use of the integration by parts identity, together with simple algebraic identities. Examples include the continued fraction of Gauss, some expansions given by Stieltjes, and expansions for several distribution functions. (Received October 3, 1957.)

THE ANNUAL MEETING IN CINCINNATI, OHIO

January 28-30, 1958

542-1. John DeCicco: *The geometry of the z-plane based on a quadratic extension* Γ *of a field K*.

The binary elements of a quadratic ring Γ are of the form $z = x + iy$, where x and y are elements of a given field K, i is not in K, but $i^2 = \alpha + i\beta$, in which α and β are two fixed elements in K. The z-plane is a vector space of two dimensions over the ground field K in which any point P is represented by a binary element $z = x + iy$ of the quadratic ring Γ. This z-plane is elliptic or parabolic or hyperbolic according as the fundamental quadratic equation $i^2 = \alpha + i\beta$, possesses no roots in K, or exactly one root in K, or two distinct roots in K. A similitude T of the z-plane is of the form $Z = a + bz$, where a and b are two fixed binary elements of the quadratic ring Γ such that $b\bar{b} \neq 0$. If $b\bar{b} = 1$, T is a motion. Some of the invariants of the similitude group G_4^* and of the motion group G_3^*, are obtained and are given geometric interpretations. The conjugate-symmetric and the symmetric inner products of two vectors in the z-plane are studied in detail. In particular, analogues of Hero's formula for the area of a triangle, the triangular inequality when K is ordered, and the law of cosines, are discussed. (Received August 26, 1957.)

542-2. Louis Auslander: *Locally affine spaces with abelian holonomy groups*.

Let M be a compact locally affine space with abelian holonomy group, $h(\Gamma)$, and fundamental group Γ. Assume that $h(\Gamma)$ contains no elements of finite order. Then Γ is the fundamental group of a compact solv-manifold S. Further, S and M are homeomorphic. Using the above result, one can also prove the following Theorem: *Let* Γ_1 *and* Γ_2 *be isomorphic fundamental groups of compact locally affine spaces* M_1 *and* M_2 *and assume the holonomy groups of* M_1 *and* M_2 *are abelian*. *Then* M_1 *and* M_2 *are homeomorphic*. (Received September 25, 1957.)

542-3. I. I. Hirschman, Jr.: On multiplier transforms. Preliminary report.

For $F(n) \in \ell^p \cap \ell^2$ $(1 < p < \infty)$ we set $F^\wedge(\theta) = \sum_{-\infty}^{\infty} F(n)e^{2\pi i n\theta}$. Let $T^\wedge(\theta)$ be a bounded measurable function on $[0,1)$. We define $(TF)(n) = \int_0^1 F^\wedge(\theta)T^\wedge(\theta)e^{-2\pi i n\theta}d\theta$. T is then a (formal) multiplier transformation. It is proved that if $T^\wedge(\theta) \in \mathrm{Lip}\,\alpha$, $0 < \alpha < 1/2$, then T is a bounded linear transformation of ℓ^p into itself for $2/(1 + 2\alpha) < p < 2/(1 - 2\alpha)$. This result is the simplest example of a general class of theorems on multipliers for Fourier series and transforms which can be established by similar methods. Among the many applications is a new and elementary proof of Stein's theorem of the L^p summability of n-dimensional Fourier transforms by Bochner-Riesz means, see Trans. Amer. Math. Soc. vol. 83 (1957) pp. 482-492. (Received September 16, 1957.)

542-4. I. I. Hirschman, Jr.: Capacity in ultraspherical algebras. Preliminary report.

A theory of fractional potentials is constructed which is related to the algebra of ultraspherical polynomials of fixed index in the same way that ordinary fractional potentials are related to Fourier series. It is proved that a restricted maximum principal is valid. From this it follows that a large part of the classical theory of capacity holds. This theory is applied to establish results for ultraspherical polynomials analagous to those obtained by Broman (Two classes of trigonometrical series, Thesis, University of Uppsala, 1947) for Fourier series. (Received September 16, 1957.)

542-5. F. B. Jones: Moore spaces and uniform structure.

Not every complete Moore space is completely regular. Hence not every complete Moore space and not every complete, regular, semi-metric space is a uniform space. An example of such a Moore space (i.e., a space satisfying Axioms 0 and 1 of Moore's Foundations) is described and certain of its other properties are pointed out. (Received October 2, 1957.)

542-6. Eckford Cohen: Representations of even functions (mod r) II. Cauchy products.

Let f(n,r) and g(n,r) be even functions (mod r). The representation theory developed in Part I is extended and applied to the Cauchy product (mod r),

$h(n,r) = \sum f(a,r)g(b,r)$, where the summation is over a,b (mod r) such that $n \equiv a + b \pmod{r}$. The representations obtained are shown, as in Part I, to form a substantial source for identities between various arithmetical functions. Many of the identities deduced appear to be of a type not previously reported. Methods for obtaining trigonometric expansions of primitive functions (mod r) are developed, and as an application a simple formula is obtained for the number $\epsilon(n,r)$ of a (mod r) such that $(a,r) = 1$ and $(n - a,r)$ is a square. Other congruence problems are considered, and certain subrings of the algebra of even functions (mod r) are discussed. (Received October 14, 1957.)

542-7. R. A. Willoughby and T. C. Chen: Computation of Eigenvalues and vectors of Hermitean matrices.

The diagonalization of a Hermitean matrix by unitary matrices is, in many respects, similar to that of a real symmetric matrix. It is desirable, however, to reduce the complex arithmetic to a minimum so as to increase speed and reduce round-off errors. It is possible to achieve this by applying the Hermitean analog of the initial stage of Givens' real symmetric method. The only change in the algorithm is that 2×2 (complex) unitary rotations are used. The resultant triple diagonal Hermitean matrix T can be further transformed into a real symmetric triple diagonal matrix R: $T = WRW^*$ where $W^* = \mathrm{diag}\left\{\exp i\sum_{j=1}^{k-1}\phi_j\right\}$, ϕ_j being the phase of the $(j,j+1)$th element of T. It follows that $R_{kk} = T_{kk}$; $R_{k,k+1} = |T_{k,k+1}| = |T_{k+1,k}| = R_{k+1,k}$. R can now be treated either by the standard Jacobi method, or by the root- and vector-finding schemes in Givens' method. In either case, existing machine codes for real symmetric matrices can be utilized, thus saving considerable coding and debugging effort. (Received October 15, 1957.)

542-8. H. J. Landau: On canonical conformal maps of multiply connected domains. I.

J. L. Walsh has recently proved the following theorem [Trans. Amer. Math. Soc. vol. 82 (1956) pp. 128-146]: Let D be a region of the extended z-plane whose boundary consists of mutually disjoint Jordan curves B_1, $B_2, \ldots, B_\mu$; $C_1, C_2, \ldots, C_\nu$, $\mu\nu \neq 0$. There exists a conformal map of D onto a region Δ of the extended Z-plane, one to one and continuous in the closures of the two regions, where Δ is defined by $1 < |T(Z)| < e^{\alpha}$, $T(Z) \equiv A(Z - a_1)^{M_1} \ldots (Z - a_\mu)^{M_\mu} / (Z - b_1)^{N_1} \ldots (Z - b_\nu)^{N_\nu}$, with α, M_i, $N_j > 0$,

$\sum M_i = \sum N_j = 1$. The locus $|T(Z)| = 1$ consists of μ mutually disjoint Jordan curves B_i^*, respective images of the B_i, which separate Δ from the a_i; correspondingly for the locus $|T(Z)| = e^{\alpha}$. In the Z-plane, the function $\log|T(Z)|/\alpha$ is the harmonic measure of the union of the curves $C_1^*, C_2^*, \ldots, C_\nu^*$ with respect to Δ, and is extendable harmonically everywhere, with the exception of one point in each of the components of the complement of Δ. Another proof, using uniformization theory, has been given by H. Grunsky [Math. Zeit. vol. 67 (1957)]. This paper presents a third and geometric proof, based on the reflection principle. Let the function $u(z)$ be harmonic in an annular region bounded by inner contour (analytic Jordan curve) Γ_1 and outer contour Γ_2, and be continuous and constant on Γ_1 with no critical points on Γ_1. Then it is shown, by repeated reflection, that the exterior of Γ_1 can be mapped conformally onto the exterior of a contour Γ so that the transform of $u(z)$ is extendable harmonically to the entire interior of Γ with the exception of one point. This map, applied successively a finite number of times, yields the canonical conformal map of Walsh. (Received October 23, 1957.)

542-9. J. L. Walsh and H. J. Landau: *On canonical conformal maps of multiply connected domains*. II.

The canonical mapping described above is generalized to domains whose boundary may contain multiple points, as follows: Let a Jordan configuration be a finite collection of (not necessarily disjoint) Jordan curves, no subset of which forms a closed cycle; that is, every connected set of constituent curves is disconnected by the removal of any two points from any one of the curves. Let D be a region of the extended z-plane bounded by a Jordan configuration consisting of Jordan curves $B_1, B_2, \ldots, B_\mu$; $C_1, C_2, \ldots, C_\nu$, $\mu\nu \neq 0$, with $\bigcup_{i=1}^{\mu} B_i$ disjoint from $\bigcup_{j=1}^{\nu} C_j$. There exists a conformal map of D onto a region Δ of the extended Z-plane, one to one and continuous in the closures of the two regions, where Δ is defined by $1 < |T(Z)| < e^{\alpha}$, $T(Z) \equiv A(Z - a_1)^{M_1} \ldots (Z - a_\mu)^{M_\mu} / (Z - b_1)^{N_1} \ldots (Z - b_\nu)^{N_\nu}$, with $\alpha, M_i, N_j > 0$, and $\sum M_i = \sum N_j = 1$. The locus $|T(Z)| = 1$ is a Jordan configuration composed of μ Jordan curves B_i^*, respective images of the B_i, which separate Δ from the a_i; correspondingly for the locus $|T(Z)| = e^{\alpha}$. (Received October 23, 1957.)

542-10. Avner Friedman: *Linear partial differential systems with an additional differential equation*.

Consider a linear partial differential system of order m, $Lu = 0$, defined in a domain D. Let $Au(x^0) = 0$ be a partial differential equation of order m at a fixed point x^0 of D, and let A be linearly independent of the equations $Lu = 0$ at x^0. One can prove that *if* L *is either elliptic or parabolic then it cannot happen that all the solutions of* $Lu = 0$ *near* x^0 *will satisfy* $Au(x^0) = 0$. If L has constant coefficients, then the proof is simple. The proof of the general case is reduced to the case of constant coefficients with the aid of fundamental solutions. If L is hyperbolic, the above theorem is a consequence of the solvability of the Cauchy problem. Using the above theorem, one can prove that *the coefficients of* L *can be solved in terms of the solutions*. (Received October 23, 1957.)

542-11. Avner Friedman: *Harnack inequality for parabolic equations and its applications to the Dirichlet problem*.

Let (1) $\sum a_{ij}\partial^2 u/\partial x_i\,\partial x_j + \sum b_i\,\partial u/\partial x_i + cu - \partial u/\partial t$ be a parabolic equation. Assume that $a_{ij}, \partial a_{ij}/\partial t, \partial^2 a_{ij}/\partial x_k \partial x_\ell, b_i, \partial b_i/\partial x_k$ and c are continuous in a domain D. Denote by M a bound on all the preceding functions. One can prove: *For any* n-*dimensional rectangle* B *interior to* D, *there exists a positive constant* A, *depending on* M *and* B, *such that for all non-negative solutions of* (1), $u(x,t) \geqq Au(x',t')$ *holds in* B, *provided* $t \geqq t'$. One can prove also an analogue of the other Harnack Theorem, and apply the *méthod de balayage* to construct a generalized solution to the Dirichlet problem. Constructing barriers, one can prove that the given (continuous) boundary values are assumed in the usual sense. (Received October 23, 1957.)

542-12. Donald Greenspan, Shozo Matsuno, and Bruno Ulrick: *On an analog solution of two point boundary value problems*.

A direct method which effectively uses analog machinery is used to solve two point boundary value problems by converting the given problem to two initial value problems which the computer easily resolves and then combines properly to yield the solution of the boundary value problem. An example is run on the REAC and the results are exhibited graphically. (Received October 25, 1957.)

542-13. Donald Greenspan: On the numerical solution of a class of elliptic differential equations. WITHDRAWN.

A study of Dirichlet type problems with elliptic differential equation: $u_{xx} + u_{yy} - (K/y)u_y = 0$, is made for K a nonzero, real constant. The difference analogue used is: $-4u_0 + u_1 + u_3 + (1 - hK/2y_0)u_2 + (1 + hK/2y_0)u_4 = 0$. Conditions are established under which the numerical solution will converge to the analytic solution. An error bound is also determined. (Received October 25, 1957.)

542-14. Donald Greenspan: On a "best" five point difference analogue of Laplace's equation.

For a rectangular grid it is shown that: $-4u_0 + (2/(1 + p^2))(u_1 + u_3) + (2p^2/(1 + p^2))(u_2 + u_4) = 0$, for $p = h/d$, and h and d the grid constants, is a best analogue for Laplace's equation in that: (a) no higher order analogue exists, and (b) every other analogue differs from this one by only a multiplicative constant. (Received October 25, 1957.)

542-15. Donald Greenspan: On nine point analogues of Laplace's equation.

It is shown that on a square grid: $-20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 = 0$ is a best analogue of Laplace's equation in that both (a) and (b) are satisfied: (a) there does not exist a higher order analogue, (b) every analogue of the same order differs from the given one by only a multiplicative constant. (Received October 25, 1957.)

542-16. Donald Greenspan: On a nine point method for the numerical evaluation of the Stokes' stream function.

A nine point method, similar to a previously developed five point method (see Mathematical tables and other aids to computation, July 1957, pp. 150-160), is described. The difference analogue: $20(-96 + 4p^2 + p^4)u_0 - 2(-192 + 44p^2 + 9p^4)(u_1 + u_3) + 2(192 - 96p + 28p^2 - 18p^3 + 5p^4 - 2p^5)u_2 + 2(192 + 96p + 28p^2 + 18p^3 + 5p^4 + 2p^5)u_4 - (-96 + 48p + 4p^2 - 6p^3 + p^4)(u_5 + u_6) - (-96 - 48p + 4p^2 + 6p^3 + p^4)(u_7 + u_8) = 0$, $p = h/y_0$, is used to convert a Dirichlet type problem to a system of linear equations. Convergence of the numerical solution to the analytical solution is established under special conditions. (Received October 25, 1957.)

542-17. Erwin Kreyszig: Properties of a class of oscillating systems having a limit cycle.

The oscillating systems under consideration correspond to differential equations of the form (1) $\ddot{q} - 2^{-1}(\mathrm{sgn}\ \dot{q})h(q)\dot{q}^2 + 2^{-1}f(q) = 0$ where $q = q(t)$ is the displacement. Let $f(q)$ be an odd polynomial satisfying $f(q) \geqq 0$ for $q > 0$. Let $h(q)$ be an even polynomial satisfying $h(q) \leqq 0$ for $-q_0 \leqq q \leqq q_0$ and $h(q) > 0$ for $|q| > q_0$ where q_0 denotes an arbitrary positive number. Then for every choice of the initial conditions the solutions of (1) correspond to oscillations which have a limit cycle, i.e. the sequence of the absolute values of the maximum amplitudes $A_1, A_2, \ldots$, has exactly one limit $Q^* > 0$. Depending on the initial conditions the aforementioned sequence decreases or increases in a monotone manner. Approximate values and upper and lower bounds of Q^* can be obtained for various types of polynomials $f(q)$ and $h(q)$. Also in the case of functions $f(q)$ and $h(q)$ satisfying weaker conditions than the above ones similar results hold. For example, if $h(q)$ is not an even function but is negative in an interval I containing the point $q = 0$ and is positive for all values $q \notin I$ then a unique limit cycle exists; in this case the absolute value of the negative maximum amplitude of this cycle is different from the positive maximum amplitude. (Received October 25, 1957.)

542-18. Eckford Cohen: A class of residue systems (mod r) and related arithmetical functions.

Let P and Q be sets of positive integers n such that $n \in P$ (or Q) if and only if every canonical divisor of n is in P (Q) and such that every $n > 0$ has a unique factorization of the form $n = ab$, $a \in P$, $b \in Q$. Define $\phi_P(r)$ to be the number of integers a (mod r) such that $(a,r) \in P$, and place $\mu_P(r) = \sum_{d|r} \mu(d)$, $r/d \in P$. The function $\mu_P(r)$ is used to prove an inversion formula for arithmetical functions with respect to the two sets P, Q. This formula is then applied to the evaluation of $\phi_P(r)$. An exponential sum generalizing $\mu_P(r)$ and $\phi_P(r)$ is also introduced and evaluated. Applications to the number of relative partitions of n (mod r) are considered. Finally, the theory of the paper is illustrated with special sets P,Q, where P consists of all k-free integers and Q is the set of k-th powers. Use is made of the theory of even functions (mod r). (Received October 28, 1957.)

542-19. L. A. Kokoris: Simple nodal noncommutative Jordan algebras.

A nodal noncommutative Jordan algebra has the form $A = F1 + N$ where F is the base field, 1 is the identity of A, and N is the set of nilpotent elements of A. By definition of noncommutative Jordan algebra A^+ is a (commutative) Jordan algebra. Define B to be the subspace of A generated by the associators $(x \cdot y) \cdot z - x \cdot (y \cdot z)$ for x,y,z in N and the dot indicates that the product is that of N^+. Also define $C_1 = B + B \cdot N$, $C_2 = B + B \cdot N + (B \cdot N) \cdot N$, $C_{k+1} = C_k + C_k \cdot N$. Since B is in N and N^+ is nilpotent, there exists a t such that $C_t = C_{t+1}$. It is shown that C_t is an ideal of N. If A is simple it follows that $C_t = 0$, $B = 0$. Therefore, A^+ is associative. The nodal algebras with A^+ associative have been determined in an earlier paper (Nodal flexible associative-admissible algebras, not yet published). (Received October 25, 1957.)

542-20. D. G. Kendall: Compactness criteria, and one-parameter semigroups of transformations.

Let $\mathcal{S}$ be a semigroup $\{T_t : t \geqq 0\}$ of bounded linear transformations from a Banach space X into itself such that (i) $T_0 = I$; (ii) $\|T_t\| \leqq M$; (iii) T_t is strongly continuous for $t \geqq 0$, and let $\mathcal{S}^0$ be the adjoint semigroup in the sense of R. S. Phillips. Let us say that "$\mathcal{S}$ determines a compactness criterion" when $\|T_t x - x\| \to 0$ uniformly (for $t \to 0$) on a bounded set A if and only if A is relatively compact. Then the following statements are equivalent: (1) $\mathcal{S}$ determines a compactness criterion; (2) $\mathcal{S}^0$ determines a compactness criterion; (3) the resolvent operator J_λ for $\mathcal{S}$ is compact for some (and then for all) $\lambda > 0$. If we identify X with the space of real-valued continuous functions having the unit circumference as support (with the customary norm) and if we identify $\mathcal{S}$ with the (semi-)group associated with the rotations, the theorem brings to light a duality between the Arzelà-Ascoli compactness criterion for C and the Marcel Riesz compactness criterion for L. (Received October 29, 1957.)

542-21. S. P. Lloyd: A sampling theorem for stationary (wide sense) stochastic processes.

Let process x have continuous covariance $R(s - t) = E\{x(s)x(t)^*\} = \int_{-\infty}^{\infty} e^{2\pi i \lambda(s-t)} F\{d\lambda\}$. Let $\mathfrak{M}$ = (closed linear manifold spanned by the collection $[x(t), -\infty < t < \infty]$), with norm $\|u\| = (E\{|u|^2\})^{1/2}$, $u \in \mathfrak{M}$, and let $\mathfrak{N}$ = (c.l.m. spanned by $[x(n\tau), -\infty < n < \infty]$), where the sample spacing $\tau > 0$ is fixed. Theorem 1: The following are equivalent: (a) $\mathfrak{N} \supset \mathfrak{M}$;

(b) $x(t) \in \mathfrak{M}$ for some irrational t/τ; (c) there exists some spectrum Λ (i.e., $F\{\Lambda'\} = 0$) which is disjoint from its translates $\Lambda \pm (m/\tau)$, $m = 1, 2, \ldots$. The proof uses an explicit formula (in the spectral representation) for projections on $\mathfrak{M}$. Theorem 2: If (c), above, holds for Λ open then $x(t) = \sum_{-\infty}^{\infty} x(n\tau)K(t-n\tau)$, where $K(t) = \int_{\Lambda} e^{2\pi i\lambda t}\, d\lambda$, and where the series is summable (C,1) in norm, and convergent in norm if Λ is a finite union of intervals. Theorem 3: Suppose (c) holds for Λ a finite union of open intervals, and suppose $R(t) = O(t^{-\alpha})$ for some $\alpha > 0$. Then $\sum x(n\tau)K(t - n\tau)$ converges with probability 1. (Received October 30, 1957.)

542-22. Frank Harary: The number of k-chromatic graphs.

A formula is obtained for the generating function whose coefficients are the number of chromatically nonisomorphic bichromatic graphs of p points and q lines. This result is then extended to the enumeration of k-chromatic graphs for any k. The counting methods are due to Pólya [Acta Math. vol. 68 (1937) pp. 145-254]. The appropriate configuration group for the bichromatic case is the cross product of symmetric groups, which is based on the pair group of a permutation group [cf. Pacific J. Math. vol. 6 (1956) pp. 57-64]. (Received October 31, 1957.)

542-23. Philip Dwinger: Completeness of quotient algebras of Boolean algebras. II.

Let A be a complete Boolean algebra and π a homomorphic mapping of A onto a Boolean algebra $\bar{A}$ with the ideal I of A as kernel. If $\{x_\xi\}$, $\xi < \alpha$, is a set of elements of A, then it is proved that $\sum \bar{x}_\xi$, $\bar{x}_\xi \in A$, does not exist in $\bar{A}$ if and only if the following condition is satisfied: If p is some element of A, such that $p \leqq x = \sum_{\xi < \alpha} x_\xi$ and $px_\xi \in I$ for every $\xi < \alpha$, then there exists an element $q \in A$ such that $q \leqq p'x$, $q \notin I$ and $qx_\xi \in I$ for every $\xi < \alpha$. (Theorem I). If I is a principal ideal of A then $\bar{A}$ is complete and π is complete. If I is an ideal generated by an infinite set of disjoint principal ideals then A/I is not complete. (Theorem II). If A is a complete atomistic Boolean algebra and I an ideal of A and K the set of the atoms of A which belong to I then the following necessary condition in order that A/I be not complete, can be derived from Theorem I: K contains an infinite disjoint family of subsets X_ξ, $\xi < \alpha$, such that $\sum X_\xi \notin I$ for every $\xi < \alpha$. Theorem I can be used in order to characterise the homomorphic mappings of a complete Boolean algebra A onto a not complete Boolean algebra. Moreover the obtained results can be extended to α-complete Boolean algebras. (Received November 5, 1957.)

542-24. J. D. Dixon: Properties of the coefficients of cyclotomic polynomials.

To investigate the properties of the coefficients of the cyclotomic polynomials $F_n(x) = \prod_{\alpha/n}(1 - x^{\alpha})^{\mu(n/\alpha)}$ the notion of order is introduced: the order $F_n(x)$ is equal to the number of distinct odd prime divisors of n. Further, $F_n(x)$ is called fundamental when n is odd, $\mu(n) \neq 0$. It has been shown (E. Lehmer, Bull. Amer. Math. Soc. vol. 42 (1936) pp. 389-392) that there exist cyclotomic polynomials of the third order with coefficients of arbitrarily large magnitude. In this paper these results are extended to prove that for any given order $\geqq 3$ there are infinitely many fundamental $F_n(x)$ with a given integer as one of its coefficients. An explicit expression for the mth coefficient of $F_n(x)$ is derived in the form of an mth order determinant. This determinant is near triangular form and has for its nonzero elements the Ramanujan sums $C_n(k)$: $1 \leqq k \leqq m$ and the first m - 1 integers. (Received November 5, 1957.)

542-25. C. N. Moore: On the frequency of prime triples.

A new variant of the classical sieve method of Eratosthenes has recently been developed, which promises to furnish fruitful attacks on many problems concerning the distribution of prime numbers that have long remained unsolved. The method has been applied to establishing the infinitude of prime pairs and obtaining a lower bound for their frequency. In the present paper it is used to obtain analogous results for prime triples. (Received November 6, 1957.)

542-26. G. T. Whyburn: Topological characterization of the Sierpinski curve.

New and greatly simplified proofs, based on more modern techniques, are given for results announced much earlier by the author but never published to the effect that any plane locally connected continuum which either (a) has no local separating point or (b) is such that each of its complementary domain boundaries is a simple closed curve and no two such boundaries intersect, is homeomorphic with the Sierpinski universal plane curve. (Received November 6, 1957.)

542-27. G. T. Whyburn: On convergence of mappings.

A study is made of sequences of mappings of two types, compact monotone and quasi-open, of X into Y where X and Y are locally connected generalized continua with a view of developing conditions for almost uniform convergence of the sequence. It is found that a modified form of the associativity condition: $\lim f_n[Fr(R)] = (\lim f_n)Fr(R)$ when applied to small regions R about points in X does yield useful implications on uniformity of convergence of the sequence. For example, if the mappings $f_n(x)$ are quasi-open, Y is cyclic and there exists a mapping f of X into Y such that for each $x \in X$ there is an arbitrarily small region R in X about x with boundary C such that almost all the sets $f_n(C)$ lie in any preassigned neighborhood of $f(C)$, then $f_n(x)$ converges almost uniformly to $f(x)$ on X. (Received November 6, 1957.)

542-28. G. T. Whyburn: Uniform convergence for monotone mappings.

Let $f_n(x)$ be a sequence of monotone mappings of X onto Y where X and Y are continua in a metric space and Y is cyclic and semi-locally connected. It is shown that if there exists a nonconstant mapping f of X into Y such that for each $x \in X$ there is an arbitrarily small open set U about x with boundary C satisfying $\lim\sup [f_n(C)] \subset f(C)$, then $f_n(x)$ converges uniformly to $f(x)$ on X and, in case Y is locally connected, f is monotone. This has the immediate consequence that if X is a regular curve and Y is cyclic, any sequence $f_n(x)$ of monotone mappings of X onto Y which converges to $f(x)$ (a nonconstant mapping of X into Y) at each point of a regularity basis for X necessarily converges uniformly on X and $f(x)$ is monotone and onto. Various other consequences and related results are obtained, some having to do with sequences of real valued functions. (Received November 6, 1957.)

542-29. G. T. Whyburn: Quasi-open mappings.

Suppose the sequence f_n of quasi-open mappings of X into Y converges almost uniformly to the mapping f, when X and Y are locally connected generalized continua. It is shown that for any $y \in Y$ and any open set V in X containing a compact component K of $f^{-1}(y)$, there exists a conditionally compact open set U in X with $K \subset \bar{U} \subset V$, a region R in Y about y and an N such that for $n > N$, U contains a component Q_n of $f_n^{-1}(R)$ so that $f_n(Q_n) = R$. This result has a variety of consequences, for example: (1) f is quasi-open, (2) If each f_n is weakly monotone or is a $1/n$-mapping and each $f^{-1}(y)$, $y \in Y$, has a compact component, f is compact and monotone, (3) If f is light, the sequence is uniformly approximately open on each compact set in A. These relate interestingly to results of J. W. T. Youngs and J. G. Hocking concerning monotone mappings on 2-manifolds and also have closure implications for certain mapping classes in the mapping space Y^X. Also for mapping generated by functions $w = f(z)$, quasi-openness is characterized in terms of a minimum modulus property of $f(z)$. (Received November 6, 1957.)

542-30. E. P. Miles, Jr.: Expansion of an arbitrary polynomial in terms of a basic set of polyharmonic polynomials.

The author and his colleague Ernest Williams recently generalized their basic set of harmonic polynomials [Proc. Amer. Math. Soc. vol. 6 (1955) pp. 191-194] to obtain a basic set of polyharmonic polynomials (i.e. solutions of $\nabla^{2m} u = 0$). For a given degree n the elements of this set are: $P^n_{a_1, a_2, \ldots, a_k} = \sum_{j=0}^{[(n-a_k)/2]} (-1)^j \cdot \binom{j + [a_k/2]}{[a_k/2]} \nabla^{2j} (x_1^{a_1} x_2^{a_2} \ldots x_{k-1}^{a_{k-1}}) x_k^{a_k+2j} / (a_k + 2j)!$ where the a_j are non negative integers such that $\sum_{j=1}^{k} a_j = n$, $a_k \leqq 2m - 1$. The totality of these polynomials with $a_k = 0$ or $a_k = 1$ gives the harmonic basic set referred to above. When $a_k = 2$ or $a_k = 3$ the polynomials are a special set of biharmonics with Laplacians giving the corresponding basic set of harmonics of degree n-2. In general the elements P^n with $a_k = 2j$ or $a_k = 2j + 1$ are polyharmonic of minimal order $j + 1$, and their Laplacians are the elements polyharmonic of minimal order j from the basic set P^{n-2}. It is sufficient to consider the expansion in terms of the P^n of a homogeneous polynomial $Q^n(x_1, x_2, \ldots, x_k) = \sum A^n_{a_1 a_2 \ldots a_k} x_1^{a_1} x_2^{a_2} \ldots x_k^{a_k}, \sum_{j=1}^{k} a_j = n$. The expansion is shown to have the form $Q^n = \sum B^n_{a_1, a_2, \ldots, a_k} \cdot P^n_{a_1, a_2, \ldots, a_k}$, where $B^n_{a_1, a_2, \ldots, a_k}$ is precisely the coefficient of $x_1^{a_1} x_2^{a_2} \ldots x_k^{a_k - 2j}$ in the polynomial $\nabla^{2j} Q^n$ where $j = [a_k/2]$. (Received November 7, 1957.)

542-31. M. Z. v. Krzywoblocki: On the superposability and self-superposability of certain partial differential equations.

The author discusses the general conditions for the superposability and self-superposability conditions for certain quasi-linear or nonlinear partial differential equations. For illustrative purposes this is applied to a simple quasi-linear partial differential system of the second order. A few different approaches are given, some of them resulting in redundant ("overspecified") systems. (Received November 7, 1957.)

542-32. T. E. Harris: *On random connections*.

Consider a grid of squares in the plane with vertices at points (x,y), where x and y are integers (positive, negative, or 0); call the sides of the squares "links." Each link has probability p of being "active," independently of other links. Let R(p) be the probability that (0,0) is contained in an infinite connected set of active links and let $p^* = \sup\{p: R(p) = 0\}$. It is known from work of Hammersley that $0.39\ldots < p^* < 0.61\ldots$. We show that $p^* \geqq 1/2$. The argument depends on using a dual grid and the fact that if $R(p) > 0$, then with probability 1 the point (0,0) is in the interior of a simple closed curve consisting of active links. (Received November 12, 1957.)

542-33. Evelyn Frank: *A note on certain determinantal equations*.

In this note criteria are given concerning the nature of the roots of determinantal equations. The equations considered here have determinants in tri-diagonal form, since a determinantal equation can be reduced to this form. (Received November 13, 1957.)

542-34. Frank Spitzer and Harold Widom. *Inversion of Toeplitz matrices*.

Let T denote the semi-infinite Toeplitz matrix $(a_{m-n})_{m,n=0,1,\ldots}$ If $\sum_{n=-\infty}^{\infty} |a_n| < \infty$, T represents a bounded operator on the space ℓ^∞ of bounded sequences $c_0, c_1, \ldots$. Let $f(\theta) = \sum a_n e^{in\theta}$. *Theorem* I: *A necessary and sufficient condition that* T *have a bounded inverse on* ℓ^∞ *is that* $f(\theta) \neq 0$ *on* $[-\pi, \pi]$ *and* $\Delta_{-\pi \leqq \theta \leqq \pi} \arg f(\theta) = 0$. In this case a generating function for the entries of T^{-1} is found explicitly. Define $X_k = \{a_{-k}, \ldots, a_0, a_1, \ldots\} \in \ell^1 (k = 0, 1, \ldots)$. *Theorem* II: *If* $f(\theta) \neq 0$ *on* $[-\pi, \pi]$, *a necessary and sufficient condition that the linear combinations of the* X_k *be dense in* ℓ^1 *is that* $\Delta_{-\pi \leqq \theta \leqq \pi} \arg f(\theta) \leqq 0$. In case $f(\theta)$ is merely bounded, T represents a bounded operator on ℓ^2. *Theorem* III: *Assume there exists a determination of log* $f(\theta)$ *in* $L_2(-\pi, \pi)$ *such that, if we write* $\log f(\theta) = \sum_{-\infty}^{\infty} b_n e^{in\theta}$, *the functions* $\sum_{-\infty}^{-1} b_n e^{in\theta}$ *and* $\sum_0^\infty b_n e^{in\theta}$ *have bounded real part*. *Then T has a bounded inverse on* ℓ^2. This assumption holds if (a) $\sum |a_n| < \infty$ and the condition of Theorem I holds; or (b) $f(\theta) = F(e^{i\theta})$ where $F(z)$ is analytic in $|z| < 1$ and continuous and nonzero on $|z| \leqq 1$; or (c) $f(\theta)$ is real and $f(\theta) \geqq m > 0$ (or $f(\theta) \leqq -m < 0$). (Received November 14, 1957.)

542-35. A. S. Householder: *A class of methods for matrix inversion*.

The methods are based upon the following: **Lemma**. *Let* P *and* CA $= P - Q$ *be nonsingular matrices of order* n, *and let* Q *be of rank* $n - r$ *at most*. *Then there exist vectors* u *and* v, *and a scalar* σ, *such that* $(I - \sigma uv^T)(P - Q) = P - R$ *is nonsingular*, *and* R *is of rank* $n - r - 1$ *at most*. If P is a matrix of known inverse, $Q = Q_1 = P - CA$, let $R = Q_2$ and repeat. If $L_i = I - \sigma_i u_i v_i^T$ one obtains ultimately $L_n(P - Q_n) = P$, or $L_n \ldots L_2 L_1 CA = P$, $A^{-1} = P^{-1} L_n \ldots L_1 C$. In general it is possible to take $v_i = e_i$, the ith column of the identity. If $C = P = I$, this choice leads to the well-known method of Jordan. With arbitrary C, if $P = I$ and $v_i = e_i$ one has a method due to Hestenes [Comm. Pure Appl. Math. vol. 8 (1955) pp. 91-92]. (Received November 18, 1957.)

542-36. Hidehiko Yamabe: *A unique continuation theorem of a parabolic equation*.

Let D be a bounded domain in d-dimensional Riemannian manifold with smooth boundary. Consider a parabolic equation $U_t = \Delta U$ under 0-boundary condition, where Δ denotes the Laplace-Beltrami operator. The results are (1) any solution u(x,t) of the above equation is analytic in t except around 0 and (2) if $u(x,t_0)$ vanishes on any open set 0 in D for a fixed t_0, it vanishes everywhere. (Received November 18, 1957.)

542-37. Paul Axt: *On a subrecursive hierarchy and primitive recursive degrees*.

The hierarchy of recursive functions h_y (of classes C_y of functions primitive recursive in h_y) and the primitive recursive degrees of Kleene (*Extension of an effectively generated class of functions by enumeration*, forthcoming) are studied. The *uniqueness property* holds at ordinal α whenever, if $y,z \in O$ (O a modified system of notations for ordinals $< \omega_1$) and $|y| = |z| = \alpha$, then $C_y = C_z$. The uniqueness property holds at $\alpha < \omega^2$ and fails at ω^2. With unmodified O, the uniqueness property fails at ω in such a way that functions of the primitive recursive degree of any general recursive predicate are definable there. For each $k > 1$, the k-recursive functions (Péter) appear in the hierarchy below the ω^{k-1} level. Many of the elementary properties of general recursive degrees of Kleene and Post (*The upper semi-lattice of degrees of recursive unsolvability*, Ann. of Math. vol. 59 (1954) pp. 379-407) hold for primitive recursive degrees. However, there are functions of the primitive recursive degree of no predicate. This result is used in the proof of analogs to Theorem 1 and its Corollaries 1-3 of Kleene and Post (op. cit.). (Received November 18, 1957.)

542-38. R. R. Phelps: Subreflexive normed linear spaces.

If E is a normed linear space let P be the set of functionals in E^* which attain their supremum on the unit cell of E. Call E subreflexive if P is norm-dense in E^*. Every reflexive Banach space is subreflexive and the nonreflexive spaces (c_0),(ℓ_1),(m), L_1 and C(X), X a compact Hausdorff space, are all subreflexive. A nonsubreflexive normed space with reflexive completion is exhibited, but the question "Is every Banach space subreflexive?" is left open. Subreflexive normed spaces are precisely those in which every bounded closed convex subset can be represented as the intersection of closed half-spaces determined by hyperplanes having a point of least norm. If E has a strongly differentiable norm, then E is subreflexive if and only if each closed bounded convex subset of E is the intersection of cells. This improves a theorem of Mazur (Studia Math. vol. 4 (1933) pp. 128-133). A space with subreflexive completion is subreflexive provided its completion has strongly differentiable norm, and any space with locally uniformly convex conjugate is subreflexive. An ℓ_p product space ($p = 1$) is subreflexive if and only if each factor space is subreflexive. (Received November 18, 1957.)

542-39. Avner Friedman: Existence and stability theorems for parabolic equations.

Let D be the topological product of an n-dimensional bounded domain R with the positive t-axis. Let L be a second order linear parabolic operator with smooth coefficients in D, and consider the equation $Lu = F(x,t,u)$ where F satisfies: $|F(x,t,u_1) - F(x,t,u_2)| \leqq \lambda|u_1 - u_2|$, for all u_i near the origin and (x,t) in D. If R is a cube, then for λ small and for small Dirichlet data, there exists a unique solution to $Lu = F$. Furthermore, the solution is stable. If the boundary data tend to zero as $t \to \infty$, then the same is true for u(x,t). The uniqueness and stability statements can also be proved for cylinders with arbitrary bases. Existence of weak solutions can be established under some assumptions on domains R which are not cubes. (Received November 18, 1957.)

542-40. J. M. Shapiro: An identity theorem for the Poisson distribution.

The following theorem is proved: Let F(x) be the Poisson distribution with parameter λ. Let H(x) be an infinitely divisible distribution function such that $H(x) = F(x)$ for $-a < x \leqq 1$ where $a > 0$. Then $H(x) = F(x)$ for all x. The proof of this theorem uses a theorem of Gnedenko and Marcinkiewicz giving necessary and sufficient conditions for sums of independent random variables to converge to the Poisson distribution. A more elementary combinatorial proof has been pointed out to the author which does not use the above mentioned theorem. However it is hoped that the more advanced methods used in the original proof will yield similar theorems for other infinitely divisible distributions. (Received November 19, 1957.)

542-41. Martin Pearl: On Cayley's parameterization. II.

The purpose of this paper is to complete the extension of the Cayley parameterization of cogredient automorphs of symmetric matrices begun in a recent paper (On Cayley's parameterization, Canadian J. Math. vol. 9 (1957) pp. 553-562). Let F be any field whose characteristic is not 2. Let P be a cogredient automorph of the symmetric matrix A whose elements are from F. Necessary and sufficient conditions that there exist a skew matrix Q such that A + Q is nonsingular and $P = (A + Q)^{-1}(A - Q)$ are that $d(P) = +1$ and that the row space of I + P is the same as the row space of A. Conversely, if $P = (A + Q)^{-1}(A - Q)$ for some skew matrix Q, then P is a cogredient automorph of A. This result extends Cayley's theorem to singular matrices over fields which are not of characteristic 2. It is shown in the paper mentioned above that the theorem as stated is not true over a field of characteristic 2. (Received November 21, 1957.)

542-42. Barry Bernstein and J. L. Ericksen: Work functions in hypo-elasticity.

In general work done per unit mass or volume in deforming a hypo-elastic material is a functional of the strain history. If it is assumed that there is a preferred strain value such that non-negative work must be done in order to change the stress from this value, then the work done per unit final volume in any deformation depends on the initial and final stresses only and the preferred stress is zero. If it is assumed that a non-negative amount of work must be done to carry a given hypo-elastic material through a deformation such that initial and final stress are the same, then the work done per unit mass in any deformation depends only upon initial and final stress. (Received November 21, 1957.)

542-43. Herbert Federer: *An integral formula*.

Suppose f is a Lipschitzian map of Euclidean n-space E_n into k-space E_k, with $n \geqq k$. For each point x where f is differentiable, let Jf(x) be the norm of the linear transformation of alternating k-vectors induced by the differential of f at x; thus Jf(x) equals the square root of the sum of the squares of the determinants of the k by k minors of the matrix of the differential of f at x. Suppose A is a Lebesgue L_n measurable subset of E_n. For each point y in E_k let u(y) be the n-k dimensional Hausdorff measure of $A \cap f^{-1}\{y\}$. Then $\int_A Jf dL_n = \int_{E_K} u dL_k$. This result is the counterpart of the classical formula for area (which applies when $n \leqq k$). (Received November 22, 1957.)

542-44. J. H. Bramble: *Continuation of biharmonic functions across circular arcs*.

This paper establishes certain reflection principles (analogous to the classical Schwarz reflection principle for harmonic functions) for biharmonic functions subject to various boundary conditions. Explicit formulae are given for the continuation of a biharmonic function w across a circular arc Q when the following conditions are satisfied. (A) $w = M(w) = 0$ on Q, (B) $M(w) = V(w) = 0$ on Q, (C) $w_r = V(w) = 0$ on Q, where $M(w) = \Delta w + (1 - \sigma)/\sigma w_{rr}$ and $V(w) = \partial(\Delta w)/\partial r + (1 - \sigma)[w_{\theta\theta r} - w_{\theta\theta}/a]/a^2$, Δ being the Laplace operator and a the radius of the circle. If σ is taken to be Poisson's ratio, then (A) and (B) correspond physically to the cases where Q is the edge of an elastic plate which is "simply supported" and "free" respectively. Condition (C) corresponds to the so-called "sliding clamped" case. (Received November 25, 1957.)

542-45. M. F. Smiley: *A remark concerning the commutativity of rings*.

In a ring R, we define $L_0(R) = R$ and, inductively, $L_{k+1}(R) = [R, L_k(R)]$. *If* $k \geqq 1$ *and every* $t \in L_k(R)$ *satisfies* $t^n = t$ *for some integer* $n > 1$, *then* $L_k(R) = 0$. *If also* $k = 1$ *or if* R *is semi-simple in the sense of McCoy* (Amer. J. Math. vol. 71 (1949) pp. 823-833), *then* R *is commutative*. This generalizes a recent result of I. N. Herstein (Canadian J. Math. vol. 9 (1957) pp. 583-586). The proof is effected by making minor changes in the proof of Herstein and establishing a lemma to the effect that if $[a, L_k(R)] = 0$ in a prime ring R, then $[a, R] = 0$. (Received November 25, 1957.)

542-46. J. C. Bradford: The Banach-Blumberg property and Blumberg's theorem.

A general topology or covering of a set is said to possess the Banach-Blumberg property whenever every subset which is locally of the first category is of the first category. It is seen that every covering with a countable basis has the property, and that if the nonempty intersection of two sets of the covering contains a set of the covering, then the covering has the B-B property. Examples are given to show that a covering may not have the B-B property, and that a covering may have the property without satisfying either of the two above sufficient conditions. A generalization of Blumberg's continuity theorem is given: A function in a metric space of homogeneous second category into a separable metric space is continuous on a dense set. (Received November 25, 1957.)

542-47. G. O. Losey: The dimension subgroups of a group.

Let G be a group, A a commutative ring with unit and Γ the group ring of G over A. Denote by Δ the ideal of Γ generated by the elements $g - 1$, for all $g \in G$. The powers, Δ^n, of Δ form a descending chain of ideals of Γ. Denote by D_n the set of all elements of G having the property: $g \equiv 1 \bmod \Delta^n$. The sequence $\{D_n\}$ is a descending central series of completely invariant subgroups of G. We call D_n the nth dimensional subgroup modulo A of G. If $A = Z$, the ring of integers, then D_n is precisely the nth term in the lower central series of G. (Received November 27, 1957.)

542-48. W. F. Davison: Cartesian products of mosaic spaces.

Theorem: The Cartesian product of a fixed mosaic space X with each mosaic space Y is a mosaic space if and only if X is locally countably compact and regular. Let X be a locally countably compact, regular mosaic space (see W. F. Davison, Convergent sequences and mosaics, Bull. Amer. Math. Soc. vol. 62 (1956) p. 180). It suffices to prove that $X \times Y$ is the quasi-compact image of a locally compact metric space. But X and Y are, respectively, quasi-compact images of suitable locally compact metric spaces P and Q under functions f and g. Write $f \times g: P \times Q \rightarrow X \times Y$ as $f \times g = (i \times g)(f \times i)$; then $f \times g$ is quasi-compact, as $f \times i$ and $i \times g$ are quasi-compact by application, respectively, of a known lemma (J. H. C. Whitehead, Note on a theorem due to Borsuk, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 1125-1132) and a modification thereof. Conversely, if X is not locally countably compact let Y be its one point countable compactification, which is a mosaic space. Then $D' = \{(x,x): x \in X\} \subset X \times Y$ is sequentially closed but not closed; therefore $X \times Y$ is not a mosaic space. Alternatively, if X is not regular let Y be the quotient space generated by the partition consisting of F and all $\{x\}$ where $x \notin F$, where F is a closed set that cannot be separated from a point $x_0 \notin F$. Since Y is a closed image of X it is a mosaic space. Then $D'' = \{(x, \{x\}): x \notin F\} \cup \{(x,F): x \in F\} \subset X \times Y$ is sequentially closed but not closed. (Received November 27, 1957.)

542-49. Alex Rosenberg and Daniel Zelinsky: Finiteness of the injective hull.

Let A be a ring with left and right descending chain condition with unit and radical N. Then if every simple ideal of A/N is finite over its centre, every irreducible left A-module can be embedded in a finitely generated injective left A-module. (Equivalently, the injective hull is finitely generated.) This is false if A does not satisfy both left and right descending chain condition. Finally, the following statements are equivalent: "There exists a ring A with both left and right descending chain conditions and an irreducible A-module that cannot be embedded in a finitely generated injective." and "There exist simple rings with minimum condition S, T such that $S \supset T$, the unit of S is the unit of T, and $[S:T]_r < \infty, [S:T]_\ell = \infty$." (Received November 29, 1957.)

542-50. R. Creighton Buck: A Weierstrass theorem for function modules.

Let X be a locally compact space, and E a locally convex real linear space. Denote by $C^*[X:E]$ the linear space of all bounded continuous functions on X into E. In a natural way, $C^*[X:E]$ can be regarded as a module over the ring $C^*[X]$ of bounded real valued continuous functions on X. A submodule A of $C^*[X:E]$ is said to be separating if $A(x_0) = \{\text{all } f(x_0) \text{ for } f \in A\} = E$ for each $x_0 \in X$; a separating submodule is fully transitive. The following analogue of the classical Stone-Weierstrass theorem is conjectured: any separating submodule of $C^*[X:E]$ is strictly dense. This is verified when E is finite dimensional, or when

X is compact and E an arbitrarily locally convex space. (Received November 29, 1957.)

542-51. H. F. Weinberger: Mean square approximation in symmetric hyperbolic systems by a finite difference method.

Let the vector u be the solution of a system $\sum_{\alpha=0}^{N} A^{\alpha} \partial u/\partial x^{\alpha} + Bu = f$ which is symmetric hyperbolic in the sense of Friedrichs (Comm. Pure Appl. Math. vol. 7 (1954) pp. 345-392), with u given at $x^0 = 0$. Let w be the solution of the corresponding finite difference problem with centered differences (Friedrichs method) on a grid with mesh size h. Let w^* be the function formed from w by making $w^* = w$ at the mesh points and constant on cubes of side h centered at the mesh points. By using the energy inequalities for both the differential and the difference equations, an explicit bound in terms of the given data and coefficients is found for the norm $\left\{\int_{x^0=T} |u - w^*|^2 dx^1 \ldots dx^n\right\}^{1/2}$. This bound approaches zero with h if the data and coefficients have continuous first derivatives. It is O(h) if they have bounded second derivatives. Under more differentiability assumptions the mean square deviation between the difference quotients of w and the corresponding derivatives of u is bounded by an explicit function that approaches zero with h. In particular, if the number of continuous derivatives is greater than $N/2 + 1$, one finds arbitrarily close upper and lower bounds for the value of u at a point by using the inequality of Sobolev (Doklady 1 (X) (1936) pp. 279-286). (Received December 2, 1957.)

542-52. R. S. Pierce: Representation theorems for certain Boolean algebras.

This paper studies Boolean algebras B which satisfy the following condition: ($D_{\alpha\beta}$) for any collection $[A_{\sigma} | \sigma \in S]$ such that $A_{\sigma} \subseteq B$ is disjointed, l.u.b. $A_{\sigma} = u$, $\overline{\overline{S}} \leqq \alpha$, $\overline{\overline{A}}_{\sigma} \leqq \beta$, and for any $b \neq 0$ in B, there exists a prime ideal P such that $b \notin P$ and $P \not\supseteq A_{\sigma}$ for all $\sigma \in S$. It is shown that a B. A. satisfies $D_{\alpha\beta}$ iff it admits a representation as a certain kind of subalgebra of an α-homomorph of a complete field. This result leads to the following corollaries: (1) if B is β-complete and satisfies $D_{\alpha\beta}$, then B is a β-subalgebra of an α-homomorph of a complete field; (2) (dual of Loomis' theorem) every $\aleph_0$-complete B.A. is an $\aleph_0$-subalgebra of an $\aleph_0$-homomorph of a complete field of sets; (3) every B.A. can be embedded as a subalgebra of an $\aleph_0$-homomorph of a complete field of sets by a mapping which preserves arbitrary joins and meets. (Received December 2, 1957.)

542-53. J. C. Wilson: Periodic solutions to an equation with rational coefficients.

We consider the equation (A): $\ddot{x} + R_1(x)\dot{x} + R_2(x) = 0$, where R_1 and R_2 are specified rational functions. With minimal conditions on the coefficients, it is shown that among the solutions of (A), there are a finite number of periodic ones, whose phase plane portraits are algebraic curves. The method employed is a straightforward application of the known theories. Questions pertaining to critical points and stability are also discussed. (Received December 3, 1957.)

542-54. B. L. Reinhart: Foliated manifolds with bundle-like metric. I.

Let M^n be a manifold with completely integrable q-form Θ and a p-form Ω such that $\Omega \wedge \Theta$ generates the cotangent space at each point. Maximal integral manifolds of Θ are called leaves. Local coordinates$(x^1,\dots,x^p,y^1,\dots,y^q)$ with (x) coordinates along the leaves may be chosen. Assume there exists a metric with local expression $g_{\alpha\beta}(x,y)\,\omega^\alpha\omega^\beta + g_{ij}(y)dy^idy^j$. Such metrics may be characterized by saying that any two transverse curves (those annihilating Ω) such that $\Pi C_1 = \Pi C_2$ have the same length, where Π is the local projection $(x,y) \rightarrow (y)$. Let M^n be complete. If M^n is simply connected, all leaves are diffeomorphic. If the foliation is regular (in the sense of Palais, Memoirs of the Amer. Math. Soc., no. 22), then M^n is a fibre space over a Hausdorff manifold. The proofs use families of transverse curves joining neighboring leaves. Example: Right invariant metric on a Lie group foliated by left cosets of any Lie subgroup. (Received December 4, 1957.)

542-55. B. L. Reinhart: Foliated manifolds with bundle-like metric. II.

Retain the definitions of I. Let a base-like differential form be one with local expression $\phi^{(y)}_{a\dots k}dy^a \wedge \dots \wedge dy^k$. The exterior derivative preserves this condition; let d' be its restriction to base-like forms. Let $*'$ and $\delta' = (-1)^{qs+q+1} *'d'*'$ be defined by using the metric induced by $g_{ij}(y)dy^idy^j$ in the bundle of base-like q-forms. Let $\Delta' = d'\delta' + \delta'd'$. Assume M^n compact and covered by a fibre space M^n with base $\tilde{B}^q$. Then the cohomology of base-like forms under d' is isomorphic to the harmonic space of Δ', is finite dimensional, and satisfies duality relations given by the isomorphism $*'$. The proof uses a compact pseudo-base for the foliation, constructed out of $\tilde{B}^q$ by identifications on the projection of a fundamental domain of the operation of $\pi_1(M^n)$ on $\tilde{M}^n$. (Received December 4, 1957.)

542-56. C. R. Karp: Formalizations of propositional languages with wffs of infinite length. Preliminary report.

Let α be a regular cardinal. Let P_α be a language with propositional symbols p_d, $d \in D$, where, in addition to the rules of formation of wffs by $\rightarrow$, $\sim$, there is a rule permitting the conjunction $[\bigwedge_{\xi<\delta} \Phi_\xi]$ to be well formed when the Φ_ξ are wffs, provided $0 < \delta < \alpha$. Here "$[\bigwedge_{\xi<\delta} \Phi_\xi]$" stands for $[\bigwedge \Phi_0 \ldots \Phi_\xi \ldots]$, where the dots indicate concatenation. The details involved in the formulation and interpretation of such languages are discussed in the author's doctoral thesis. Let $\mathcal{P}_\alpha$ be a logical system based on P_α where the rules of inference are modus ponens and conjunction (from $\vdash \Phi_\xi$ for $\xi < \delta < \alpha$ infer $\vdash \bigwedge_{\xi<\delta} \Phi_\xi$), and proofs may be sequences of wffs of order type less than α. Take as axioms those of the ordinary propositional calculus together with instances of schemata $\bigwedge_{\xi<\delta}[\Psi \rightarrow \Phi_\xi] \rightarrow . \Psi \rightarrow [\bigwedge_{\xi<\delta} \Phi_\xi]$ and $[\bigwedge_{\xi<\delta} \Phi_\xi] \rightarrow \Phi_\xi$ for $\xi < \delta$, $0 < \delta < \alpha$. Then $\mathcal{P}_{\omega_1}$ is semantically complete. But for $\alpha > \omega_1$, if D is infinite, $\mathcal{P}_\alpha$ is not semantically complete. If, however, we added to $\mathcal{P}_\alpha$ axioms and rules of inference so that $\vdash \sim \bigwedge_{\xi<\delta} \bigvee_{\nu<\delta} \Phi_{\xi\nu}$ whenever $0 < \delta < \alpha$ and for every $f \in \delta^\delta$, the set $\{\Phi_{\xi f(\xi)} ; \xi < \delta\}$ includes a wff and its negation, then the system would be complete. If α is inaccessible, it suffices to add as axioms instances of choice schemata $\bigwedge_{\xi<\delta} \bigvee_{\nu<\delta} \Phi_{\xi\nu} \rightarrow \bigvee_{\theta<\gamma} \bigwedge_{\xi<\delta} \Phi_{\xi f_\theta(\xi)}$ where $\{f_\theta ; \theta < \gamma\} = \delta^\delta$.

(Received December 4, 1957.)

542-57. C. R. Karp: Formalizations of functional languages with wffs of infinite length. Preliminary report.

For notation see the preceding abstract. Let α, β be cardinals, $\alpha \geqq \beta \geqq \omega$, α regular. Given a supply of function symbols and at least α individual symbols v_i, $i \in I$, let $F_{\alpha\beta}$ be a functional language where, in addition to the rules of formation of wffs of P_α, there is a rule permitting $[\forall_{\xi<\epsilon} v_{f(\xi)} \Phi]$ to be a wff when Φ is a wff, $0 < \epsilon < \beta$, $f \in I^\epsilon$. Here "$[\forall_{\xi<\epsilon} v_{f(\xi)} \Phi]$" stands for $[\forall v_{f(0)} \ldots v_{f(\xi)} \ldots \Phi]$ where the dots indicate concatenation. The details involved in the formulation and interpretation of such languages are discussed in the author's doctoral thesis. Let $\mathcal{F}_{\alpha\beta}$ be a logical system based on $F_{\alpha\beta}$ where the rules of inference are modus ponens, conjunction, and generalization (from $\vdash \Phi$ infer $\vdash [\forall_{\xi<\epsilon} v_{f(\xi)} \Phi]$ whenever $0 < \epsilon < \beta$, $f \in I^\epsilon$), and formal proofs may be sequences of wffs of order type less than α. Take as axioms instances of the schemata of $\mathcal{P}_\alpha$ together with the obvious modifications of the usual schemata for universal quantification. Then $\mathcal{F}_{\omega_1 \omega}$ is semantically complete, but $\mathcal{F}_{\omega_1 \omega_1}$ is not, and neither are systems $\mathcal{F}_{\alpha\beta}$ where $\alpha > \omega_1$. Additional rules of inference are given to complete these systems. In case α is inaccessible, it suffices to add instances of the choice schemata as axioms and a rule of inference permitting passage from certain conjunctions to universal quantifications. (Received December 4, 1957.)

542-58. C. R. Karp: Formalisms for P_α, $F_{\alpha\beta}$ and α-complete Boolean algebras. Preliminary report.

For notation see preceding abstracts. Languages P_α, $F_{\alpha\beta}$ can be interpreted in certain α-complete Boolean algebras along lines of Henkin (Fund. Math. vol. 37 (1950) pp. 63-74). We call a set Γ of wffs algebraically satisfiable if in some such interpretation every wff of Γ assumes the value 0. Then for $\mathcal{P}_\alpha$, $\mathcal{F}_{\alpha\beta}$, Γ is consistent (i.e., if $\Phi_\xi \in \Gamma$ for all $\xi < \delta$, $0 < \delta < \alpha$, then $\nvdash \sim [\bigwedge_{\xi<\delta} \Phi_\xi]$) if and only if it is algebraically satisfiable. Using this result along with an analogue of the Skolem-Lowenheim Theorem which holds for $F_{\alpha\beta}$, syntactical questions about extensions of $\mathcal{P}_\alpha$, $\mathcal{F}_{\alpha\beta}$ are seen to have equivalents in the theory of α-complete Boolean algebras. For example, let us call a α-complete Boolean algebra $\mathcal{B}$ weakly (γ,α)-distributive if $\prod_{i\in I} \sum_{j\in J_i} b_{ij} = 0$ whenever $\{b_{ij};\ i \in I,\ j \in J_i\}$, $\overline{\overline{I}} < \gamma$, $\overline{\overline{J}}_i < \alpha$, is a doubly indexed set of elements of the algebra such that $\{b_{if(i)};\ i \in I\}$ contains a complementary pair for every $f \in \mathcal{P}_i J_i$. Then $\mathcal{B}$ is weakly (α,α)-distributive if and only if it is an α-homomorphic image of an α-field of sets. $\mathcal{B}$ is weakly (∞,α)-distributive if and only if it is isomorphic to a α-field of sets. Compare results of Chang (Bull. Amer. Math. Soc. vol. 61 (1955) p. 325) and Smith (Bull. Amer. Math. Soc. vol. 61 (1955) p. 134). (Received December 4, 1957.)

542-59. L. E. Payne and H. F. Weinberger: Bounds in the exterior Neumann problem.

Let $\vec{g}$ be a piecewise differentiable vector field defined in the exterior D of a bounded surface B in N-space ($N \geqq 3$). Let $\operatorname{div} \vec{g} = O(r^{-2})$ as $r \to \infty$, and suppose that $\operatorname{div} \vec{g} - |\vec{g}|^2$ is uniformly bounded below. Furthermore let $\vec{g} \cdot \vec{\nu}$ be positive, where $\vec{\nu}$ is the unit normal on B pointing into D. Let u be any Dirichlet integrable function in D such that $r^{-2}u^2$ and $r^2(\Delta u)^2$ are integrable. The following inequality is established: $\int_o (\operatorname{div} \vec{g} - |\vec{g}|^2) u^2 dv + \oint_B (\vec{g} \cdot \vec{\nu}) u^2 ds \leqq D(u) \leqq \int_D (\operatorname{div} \vec{g} - |\vec{g}|^2)^{-1} (\Delta u)^2 dv + \oint_B (\vec{g} \cdot \vec{\nu})^{-1} (\partial u / \partial \nu)^2 ds$. If Δu is given in

D and the normal derivative $\partial u/\partial \nu$ on B, this inequality serves to bound the Dirichlet integral D(u) as well as the integrals of u^2 over D and B. In practice, if Δw and $\partial w/\partial \nu$ are given, one chooses a function ϕ such that, if $u = w - \phi$, the right-hand side of the inequality is small. This serves to give upper and lower bounds for D(w) and, by use of a suitable parametrix, for the value of w at any interior point of D. The method is easily extended to more general second order elliptic operators. (Received December 4, 1957.)

542-60. K. Padmavally: A mixed boundary value problem.

A solution of the partial differential equation $\phi_{xx} + \phi_{yy} = \phi - 1$ is sought, which $\rightarrow 1$ at infinity, and vanishes on the closed segment (0,1) of the x axis. These conditions are satisfied by the density ϕ of neutrons in a reactor, assumed to be infinite, in which the portion $0 \leqq x \leqq 1$ of the y-plane is occupied by a control rod. The symmetry of the problem about the x-axis reduces it to determining a solution ϕ of the differential equation in the upper half-plane, which vanishes on the closed segment (0,1) of the x-axis, $\rightarrow 1$ at infinity, and has zero normal derivative on the rest of the boundary. Using a suitable Neumann function this problem is reduced to the solution of a Fredholm equation of the first kind. Then using a formula due to W. Schmeidler (Integralgleichungen mit Anwendungen in Physik und Technik, p. 57) it is further reduced to the solution of a Fredholm equation of the second kind. (Received December 5, 1957.)

542-61. J. A. Nohel: Stability and instability in reactor dynamics. Preliminary report.

A reactor is said to be dynamically stable or unstable according as to whether the equilibrium solutions of the dynamic reactor equations possess this property or not. A typical system of equations has the form (1) $x' = [A + B(t)]x + f(t,x) + g(t)$, $' = d/dt$, where t is the time and x, f, g are real vectors with n components, and A is a real constant $n \times n$ matrix, while B(t) is a real variable $n \times n$ matrix. f(t,x) is analytic and of order $O(|x|)$ for small $|x|$ uniformly int. The liquid metal fuel reactor gives rise to the system (1) with $n = 12$. If $g(t) = 0$, $x = 0$ is a solution and the stability of this solution is studied with $g = 0$, under various physically reasonable hypotheses involving A and B. In light of these results it is possible to obtain the behaviour of the solutions of the full system (1), if g is suitably restricted. Generalizations of classical results on stability of solutions of differential equations are required in certain cases. The results show that physically relevant forms of B(t) can lead to instability. (Received December 5, 1957.)

542-62. Joshua Chover: *Conditions on the realization of prediction by measures*.

The following theorem is proved: Let x_t be a nondeterministic second-order-stationary stochastic process, $-\infty < t < \infty$; with $Ex_t = 0$ and $r(t) = Ex_s\bar{x}_{s+t}$. Let $-\infty \leqq a < b < c < \infty$; and suppose that for each t in $b < t < c$ there is a Radon measure m^t on the interval $a \leqq s \leqq t$ such that the linear least-squares prediction "$P_a^t x_c$" of x_c given x_s, $a \leqq s \leqq t$, has the form $P_a^t x_c = \int_a^t x_s dm^t(s)$. Let $D_0^{\pm} r$ be the one-sided derivatives of r at $t = 0$ (if they exist). Then none of the following can be true: (I) $D_0^{\pm} r$ exist, and are both zero; (II) $D_0^{\pm} r$ exist, and $m^t(\{t\}) = 0$ for $b < t < c$; (III) $D_0^{\pm} r$ exist, and $\lim_{s \uparrow t} (t - s)^{-1} \cdot \|x_t - P_a^s x_t\|^2 = 0$ for $b < t < c$. (Received December 5, 1957.)

542-63. I. N. Herstein and M. F. Ruchte: *Semi-automorphisms of groups*.

If G is a group, by a semi-automorphism of G is meant a 1-1 mapping ϕ of G onto itself so that $\phi(aba) = \phi(a)\phi(b)\phi(a)$ for all $a, b \in G$. Dinkines had shown that for alternating and symmetric groups every semi-automorphism is either an automorphism or an anti-automorphism. In this paper it is shown that if G is a simple group (finite or infinite) with an element of order 4 then every semi-automorphism of G is either an automorphism or an anti-automorphism. As corollaries Dinkines' results are obtained. The proof proceeds by reducing from a general semi-automorphism to one which commutes with all inner automorphisms. The proof uses the Burnside problem for exponent 4. (Received December 5, 1957.)

542-64. P. M. Swingle: *Connected towers of Wada*.

Let D_g ($g = 0,1,2,\ldots$) be a connected domain of Wada such that each D_g is contained and dense in D_{g-1} ($g \neq 0$) and B_g is the boundary of D_g. We call $\bigcup D_g$ a connected domain tower of Wada and $\bigcup B_g$ a connected boundary tower of Wada. The following are typical of the results obtained. Let $m \geqq 3$ and the euclidean $E_m \supset D_0$ whose boundary B_0 is a hereditarily indecomposable continuum; let D_g ($g \neq 0$) be a Wilder-Wada domain (Wilder, *Domains and their boundaries in* E_m, Math. Ann. vol. 109 (1933) Theorem 1, p. 275): then each

B_g is the disjoint sum of the hereditarily indecomposable B_0 and its locally and arc-wise connected coating $B_g - B_0$; if $\bigcap D_g \not\supset$ a region, $\bigcap D_g$ is an indecomposable connected set with an uncountable number of disjoint composants each dense in $C = \bigcap D_g$ and, if K is any connected subset of C such that $\overline{K} \neq \overline{C}$, then $C \supset \overline{K}$. There exists in the Hilbert cube I_ω an infinite-dimensional locally and arc-wise connected set $\bigcup B_g$ such that each B_g and each of its coatings $B_g - B_h$, for each $h < g$, is locally and arc-wise connected. Also cases where D_g is disconnected are considered, some giving that $\bigcap D_g + B_0$ is an n-indecomposable connected set with uncountably many composants. (Received December 5, 1957.)

542-65. P. M. Swingle: Hereditarily indecomposable towers and related sets.

Let $m \geqq 3$ and the euclidean E_m contain the connected domain D_0 with hereditarily indecomposable boundary B_0; let $D_g \supset D_{g+1} (g = 1,2,\ldots)$ where D_g is a connected domain with boundary B_g. Then the following is typical of the results obtained; let further B_g be the disjoint sum of B_{g-1}, W_g and C_g, where W_g is a hereditarily indecomposable coating of B_{g-1} and C_g is an arc-wise and locally connected coating of B_{g-1}; each C_g is the part of the boundary in D_{g-1} of a Wilder-Wada connected domain D'_g and W_g is the part of the boundary in D'_{g-1} of the connected domain D''_g such that $D'_g = D''_g + W_g + D_{g+1}$ and $D_g = D'_g + C_g$: then $\bigcap D_g$ is an indecomposable connected set, if $\bigcap D_g \not\supset$ a region, with uncountably many composants each dense in $\bigcap D_g$ and $(C_1 + C_2 + \ldots + C_k)$ and $\bigcup C_g$ are both arc-wise and locally connected; $B_0 + (W_1 + W_2 + \ldots + W_k)$ and $\bigcup W_g$ are both hereditarily indecomposable connected sets. Here C_g is obtained by Wilder's construction (Math. Ann. vol. 109 (1933) Theorem 1, p. 275) and W_g by a modification of Bing's construction in (Higher dimensional hereditarily indecomposable continua, Trans. Amer. Math. Soc. vol. 71 (1951) Theorems 1-3, pp. 268-270). (Received December 5, 1957.)

542-66. P. M. Swingle: Connected sets of Vitali.

The imbedding space is the euclidean E_m, $m \geqq 2$, and $D \subset E_m$, is a connected domain. See Kuratowski (Toplogie, vol. 1, p. 59) or Sierpinski (Fund. Math. vol. 34 (1947) p. 160) for Vitali's construction of Lebesgue non-measurable sets; if C is obtained by this construction, except for a set of measure zero, we say C is a set almost of Vitali. A connected set C almost of Vitali neither contains nor is contained in an $(m - 1)$-plane, but for C disconnected this is false. If C is a connected set almost of Vitali, it is shown that C can be: locally connected; the imbedding set of a hereditarily indecomposable m'-dimensional continuum, $m' \leqq m - 2$; for $m = 2$ a biconnected set with dispersion point; the imbedding set of a widely connected set; the imbedding set of an irreducible connected set between two points; an n-indecomposable connected set; a finitely-indecomposable connected set: for all except the imbedding cases D is the disjoint sum of C, the rational translations C_i $(i = 1,2,\ldots)$ of C, and of a set M of measure zero. There exists in D an indecomposable continuum I of nonzero measure which is the disjoint sum of M of zero measure and of the C_i $(i = 1,2,\ldots)$ where each C_i is an indecomposable connected set dense in I and at least one C_i is Lebesgue nonmeasurable. (Received December 5, 1957.)

542-67. Walter Rudin and Karel deLeeuw: The extreme points of the unit ball in H_1.

H_1 is the class of all functions which are analytic in the open unit disc U and for which $\|f\| = \lim_{r\to 1}(2\pi)^{-1}\int_0^{2\pi}|f(re^{i\theta})|d\theta < \infty$. With this norm, H_1 is a Banach space, and is the dual space of C/A, where C is the space of all continuous functions on the unit circle, and A consists of those members of C which are boundary values of uniformly continuous analytic functions in U. The Krein-Milman theorem thus implies that S (the set of all $f \in H_1$ with $\|f\| \leqq 1$) has extreme points. However, the following more precise information is obtained directly: Theorem 1. (a) f is an extreme point of S if and only if $\|f\| = 1$ and $f(z) = \exp\left\{(2\pi)^{-1}\int_0^{2\pi}(e^{i\theta} - z)^{-1}(e^{i\theta} + z)\log|f(e^{i\theta})|d\theta\right\}$. (b) If $\|g\| = 1$, there exist extreme points f_1, f_2 of S such that $2g = f_1 + f_2$. (c) If $\|g\| < 1$, then g is a convex combination of two extreme points of S. Next, let $T(\neq 0)$ be a bounded linear functional on H_1, and let E_T be the set of all extremal functions of T, i.e., the set of all $f \in H_1$ such that $\|f\| = 1$ and $Tf = \|T\|$. (For some T, $E_T = 0$.) Theorem 2. (a) If $E_T \neq 0$, then E_T contains an extreme point of S. (b) E_T may contain more than one function; in that case, $1/f$ is unbounded in U for every $f \in E_T$, and for every $z_0 \in U$ there is an $f \in E_T$ with $f(z_0) = 0$. (Received December 6, 1957.)

542-68. James Eells, Jr.: A generalization of the Gauss-Bonnet theorem.

Let S_p = bundle of p-frames on the smooth orientable Riemann n-manifold X. For each $2 \leqq r \leqq n$ we construct forms Ω^r on S_{n-r} and Φ^{r-1} on S_{n-r+1}

which are polynomials in the connection and curvature forms of X; for any r-dimensional submanifold V of X these forms determine the Gauss and geodesic curvature forms of V. By taking suitable (n-p)-frames f_p partially defined on X we obtain forms $f_r^*\Omega^r$ and $f_{r-1}^*\Phi^{r-1}$(with characteristic cycles as singularities) satisfying $f_r^*\Omega^r = df_{r-1}^*\Phi^{r-1}$. By evaluating the difference (a generalized residue) of these integrals on any integral chain and its boundary (avoiding the singularities) we obtain an integral r-cochain whose cohomology class is precisely the rth Stiefel-Whitney class of X; when X is compact and $r = n$ we have the Gauss-Bonnet Theorem. The proof is based on a generalization (due to Allendoerfer-Eells) of de Rham's Theorem, giving the integral cohomology of X in terms of residues of pairs of forms with appropriate singularities. Applications are made to integral formulas for relative characteristic classes. We also obtain the mod 2 cohomology of Grassmannians in terms of residues of pairs of Maurer-Cartan polynomials. (Received December 6, 1957.)

542-69. P. H. Maserick: Set functions and translation half-rings in the plane.

A planar half ring (Von Neumann, Functional operators, 1950) is a family $\mathcal{A}$ of sets in the plane satisfying (i) $R, R' \in \mathcal{R}$ implies $R \cap R' \in \mathcal{R}$ (ii) $R, R' \in \mathcal{R}$, $R \subset R'$ implies there exists a finite chain $R_1, R_2, \ldots, R_k$, $R_i \in \mathcal{R}$ for $i = 1, 2, \ldots, k$ such that $R = R_1 \subset R_2 \subset \ldots \subset R_k = R'$ and $R_j - R_{j-1} \in \mathcal{R}$ for $j = 2, 3, \ldots, k$. If the additional assumption (iii) $R \in \mathcal{R}$, R^* a translate of R implies $R^* \in \mathcal{R}$, R is called a planar translation half ring (t.h.r.). $\mathcal{R}$ is nondegenerate if it contains an element with a nonvoid interior. It is shown that the set of all convex polygons oriented with respect to n directions is a planar t.h.r. The main result is the proof of the converse of the above, namely that if $\mathcal{P}$ is a nondegenerate planar t.h.r. of convex sets, then $\mathcal{P}$ is a family of convex polygons. It is further shown that $\mathcal{P}$ has a finite number of directions if and only if there is a $P \in \mathcal{P}$ with a maximum number of sides and also for every two directions d_1, d_2 of $\mathcal{P}$ there is a nondegenerate rhombus $R \in \mathcal{P}$ which is oriented with respect to d_1 and d_2. From this latter result it is proved that each $P \in \mathcal{P}$ has the same number of sides if and only if $\mathcal{P}$ is a set of oriented parallelograms. (Received December 6, 1957.)

542-70. Gilbert Helmberg: Generating sets of elements in compact groups.

Let G be a compact group whose space is T_1, μ its normalized Haar measure and C(G) the convolution-algebra of all continuous complex valued functions on G. For any nonvoid subset M of G let F(M) be the set of all functions of C(G) which are invariant under right translation $[f_m(x) = f(xm)]$ by all elements $m \in M$. If H_M is the closed subgroup of G generated by M then $F(M) = F(H_M)$. If H is any closed subgroup of G then $M \subset H$ iff $F(M) \supset F(H)$. F(H) is a closed left ideal in C(G), finitedimensional iff $\mu(H) > 0$, twosided iff H is normal. Let $R^{(\lambda)}$ $(\lambda \in L)$ be a complete system of inequivalent irreducible unitary representations of G of degree r_λ. Let $f^{(\lambda)}(H)$ be the multiplicity with which the identity-representation of H is contained in $R^{(\lambda)}$ restricted to H and for any nonvoid subset M of G $f^{(\lambda)}(M) = f^{(\lambda)}(H_M)$. Investigating the intersections of F(H) with the twosided ideals in C(G) spanned by the r_λ^2 entries of $R^{(\lambda)}$ it is seen that $1:\mu(H) = \sum_\lambda r_\lambda f^{(\lambda)}(H)$ (the sum diverges if $\mu(H) = 0$). H is normal iff $f^{(\lambda)}(H)$ has value 0 or r_λ for all $\lambda \in L$. $a \in H_M$ iff $f^{(\lambda)}(M) = f^{(\lambda)}(a \cup M)$ for all $\lambda \in L$. M generates G iff $f^{(\lambda)}(M) = 0$ for all $R^{(\lambda)}$ except the identity-representation. For finite M $r_\lambda - f^{(\lambda)}(M)$ is the rank of a rectangular matrix. (Received December 9, 1957.)

542-71. T. G. Ostrom and A. Wagner: Transitive affine planes and doubly transitive projective planes.

The main results of this paper are (1) A finite affine plane which is transitive on lines is a Veblen-Wedderburn plane except, possibly, when n is the square of an odd composite number. (2) All finite doubly transitive projective planes are Desarguesian. Result (1) depends upon a Lemma of Gleason (Finite fano planes, Amer. J. Math. vol. 78 (1956) pp. 797-807) to the effect that if, for all points P on the line l the groups of elations with center P and axis l have the same nontrivial order, then the little theorem of Desargues holds on l. The existence of elations with axis l is shown by considering Sylow p-groups of the collineation group, where p is a factor of n or, if n is not a square, using the methods of Ostrom in Double transitivity in finite projective planes, Canadian J. Math. vol. 8 (1956) pp. 563-567. Result (2) follows from (1) except when n is an odd square which might not be a prime power. With double transitivity, we show that n must be a prime power by showing the existence of a Desarguesian subplane of order n_1, where n is some power of n . Received December 9, 1957.)

542-72. J. C. C. Nitsche: Isolated singularities of solutions of regular variational problems.

Consider the regular variational problem (*) $\delta \iint F(p^2 + q^2) dxdy = 0$. It is assumed that the function $F(w)$, $w = p^2 + q^2$, belongs, for all non-negative values of w, to class C^2 and that $F'(w) > 0$ and $1 + w\lambda(w) > 0$. Here $\lambda(w) = 2F''(w)/F'(w)$. Let a solution $z(x,y)$ of the Euler equation $a(p,q)\cdot r + 2b(p,q)\cdot s + c(p,q)\cdot t = 0$ of (*) be given, single valued and of class C^2 in a punctured disc D_R: $0 < x^2 + y^2 \leqq R^2$. The following theorem is proved: A necessary and sufficient condition for the singularity of $z(x,y)$ at $x = y = 0$ to be removable is the convergence of the integral $\int^{\infty}((1 + w\lambda(w))/w)dw$. (In case of the minimal surface one has $1 + w\lambda(w) = (1 + w)^{-1}$; hence isolated singularities are removable, see L. Bers, Ann. Math. vol. 53, 1951. A sufficiency proof was given by R. Finn, Trans. Amer. Math. Soc. vol. 75 (1953).) (Received December 9, 1957.)

542-73. P. J. McCarthy: Note on indefinite ternary genera of more than one class.

Using the methods of the author's previous paper (Duke Math. J. vol. 24 (1957) pp. 19-24) it is shown that if f is a properly primitive indefinite ternary quadratic form with integral matrix and properly primitive reciprocal form, then the order of f has a genus of more than one class in the following cases: (1) $\Omega = 2^{\omega}\Omega_1^2 n$ and $\Delta = -2^{\delta}\Delta_1^2 n$ where Ω_1, Δ_1 are odd, n is odd, positive and squarefree, ω and δ both even, $\omega + \delta \geqq 2$ when $n \equiv 5 \pmod 8$ and $\omega + \delta \geqq 4$ when $n \equiv 3 \pmod 4$; (2) $\Omega = 2^{\omega}\Omega_1^2 nu$ and $\Delta = -2^{\delta}\Delta_1^2 n$ where Ω_1, Δ_1 are odd, n and u odd and positive, nu squarefree, $\Delta_1 \equiv 0 \pmod u$, ω and δ both even, $\omega + \delta \geqq 2$ when $nu \equiv 5 \pmod 8$ and $\omega + \delta \geqq 4$ when $nu \equiv 3 \pmod 4$. (Received December 9, 1957.)

542-74. R. H. Bing: A surface S is tame in E^3 if E^3 - S is locally simply connected at each point of S.

Suppose S is a closed set in E^3 which is a 2-manifold. $E^3 - S$ is locally simply connected at a point p of S provided that for each neighborhood U of p in E^3 there is a neighborhood V of p such that each closed curve in $V\cdot(E^3 - S)$ can be shrunk to a point in $U\cdot(E^3 - S)$. It is shown that S is tame (there is a homeomorphism of E^3 onto itself taking S onto a polyhedron) if and only if $E^3 - S$ is locally simply connected at each point of S. The proof of this result makes use of the approximation theorem for surfaces (a surface can be homeomorphically approximated by a polyhedral surface) and the result that a surface is tame if it can be homeomorphically approximated from both sides. (Received December 9, 1957.)

542-75. Juris Hartmanis: *Algebra of linear sequential coding networks*.

Linear multivalued sequential-coding networks are circuits whose input and output are synchronized sequences of non-negative integers less than some fixed integer m. The output depends linearly on the present input and a finite number of previous inputs and outputs. The transfer characteristics of such a network are described by a ratio of polynomials in the delay operator, where the multiplication and addition are performed with respect to the fixed modulo m. An algebraic theory of the delay polynomials is obtained in terms of their null sequences. It is also shown that a rational transfer function can be realized if the denominator contains a constant term prime to m and explicit constructions are given. It is noted that a coding network is stable if the polynomial in the denominator of the transfer function has no null sequences. Thus any non-trivial coding network or its inverse is unstable if one works modulo a prime. If the modulo is not a prime, stable networks with stable inverses are constructed. Finally it is indicated how polynomials with no null sequences can be used to simplify the construction of coding networks. (Received December 9, 1957.)

542-76. E. E. Grace: *Totally nonconnected im kleinen continua.*

If a connected topological space T is not connected im kleinen at the point p of T, then there is some open set D containing p such that p is not an interior point of the p-component of D. Related results are studied for a certain class of topological continua (Baire topological continua) that are not connected im kleinen at any point of a particular type of somewhere dense subset. The following is a corollary of one of the main theorems. *Let* M *be a complete, perfectly separable, metric continuum*. *Then* M *is not connected im kleinen at any point of a dense inner limiting* (i.e. G_δ) *subset of* M *if and only if there is a dense open subset of* M, *no component of which contains an open set*. That similar results can not be gotten for continua that are not locally connected at any point is shown by an example of a bounded plane continuum that is connected im kleinen at each point of a dense inner limiting subset but does not contain a nondense open connected subset. (Received December 9, 1957.)

542-77. Isidore Fleischer and Anthony Kooharian: *On the statistical treatment of stochastic processes*.

If $x(t,w)$ is a stochastic function defined on $T \times \Omega$ and such that (a) $x(t, \ ) \in L_2(\Omega)$ for each $t \in T$, (b) $r(s,t) \equiv E\{x(s,w)x(t,w)\}$ is continuous on $T \times T$, (c) $\int_T r(t,t)dt < \infty$, then by the general representation theory of Karhunen $x(t,w) \sim \sum_{k=1}^{\infty} (z_k(w)/(\lambda_k)^{1/2}) \phi_k(t)$ where the series converges in the mean on Ω for each $t \in T$. It is shown that the Karhunen representation converges (Ω weakly) in the mean on T; i.e., the inner product operation in $L_2(\Omega)$ can be performed term by term in the series so far as convergence in the mean on T is concerned. This result is used to apply statistical techniques such as hypothesis testing and parameter estimation to stochastic processes (c.f. U. Grenander, *Stochastic processes and statistical inference*, Arkiv für Mat. vol. 1 (1950) pp. 195-277). Examples are included of hypothesis testing for the covariance function and parameter estimation of the mean of normally distributed stochastic processes. (Received December 9, 1957.)

542-78. M. S. Klamkin: *On the solution of a nonlinear integral equation arising in a study of advection fogs*.

The visibility V of an advection fog is given empirically by the formula, $cV^d = \int_0^\infty r^3 n(r_m - r)dr$, where $n(r_m - r)$ is the distribution function for the number of particles of different sizes. From theoretical considerations, $eV^{-1} = \int_0^\infty r^2 n(r_m - r)dr$. Consequently, the distribution function $n(r_m - r)$ must satisfy the nonlinear integral equation, $\int_0^\infty r^3 n(r_m - r)dr = a[\int_0^\infty r^2 n(r_m - r)dr]^b$, where a and b are independent of r_m. Extraordinarily, this nonlinear equation can be solved elegantly, leading to the unique solution $n(r) = A[r + B]^k$, where A, B and k are related to a and b (we assume that n(r) is differentiable). (Received December 9, 1957.)

542-79. C. J. Burke and Murray Rosenblatt: *A Markovian function of a Markov chain*.

Let X(t) be a finite state continuous parameter Markov chain with stationary transition probability function P(t), $0 < t < \infty$. Let P(t) be continuous and such that $\lim_{t \downarrow 0} P(t) = I$. Conditions under which a function $Y(t) = f(X(t))$ of X(t) is Markovian are studied. Given f, necessary and sufficient conditions are given for Y(t) to be Markovian, whatever the initial probability distribution of X(t). The states i of X(t) on which f equals some fixed constant are collapsed into a single state of the process Y(t). Call the collapsed sets of states S_i, $i = 1,\ldots,r$. Let $p_{jk}(t) = P[X(t) = k | X(0) = i]$, $t > 0$. The conditions read as follows: Given any set of states S_i either (a) $p_{jk}(t) = 0$ for all $j \notin S_i$ and $k \notin S_i$ or else (b) for each $\ell = 1,\ldots,r$, $\sum_{k \in S_\ell} p_{jk}(t)$ is a constant for all $j \in S_i$. (Received December 9, 1957.)

542-80. R. G. Laha: On a factorization theorem in the theory of analytic characteristic functions.

The following theorems are proven: Theorem 1. Let $\phi_1(t),\ldots,\phi_n(t)$ be characteristic functions of some nondegenerate distributions and $\alpha_1, \alpha_2, \ldots, \alpha_n$ be some positive numbers. Let the functions $\phi_j(t)$ satisfy the equation $\prod_{j=1}^{n} \{\phi_j(t)\}^{\alpha_j} = \Phi(t)$ for all real t in a certain neighborhood $|t| < \delta$ $(\delta > 0)$ of the origin, where $\Phi(z)$ is an analytic characteristic function. Then each of the factors $\phi_j(z)$ is also an analytic characteristic function which is regular at least in the strip of regularity of $\Phi(z)$. Theorem 2. Under the same conditions as in Theorem 1, let $\Phi(z)$ be an entire characteristic function of some finite order ρ. Then each of the factors $\phi_j(z)$ is also an entire characteristic function of order not exceeding ρ. (Received December 9, 1957.)

542-81. Anil Nerode and H. Shank: Theorems of Kirchhoff type.

Let C be the real inner product space of 1-chains with orthonormal basis the set of branches of a connected oriented finite graph G. If T is a maximal tree of G, define a linear transformation $\overline{T}: C \to C$ for each branch b by letting $\overline{T}b$ be the 1-cycle in $T \cup \{b\}$ in which b occurs with coefficient 1, 0 if none exists. Theorem: suppose $R: C \to C$ is a linear transformation such that for each branch b, $Rb = r_b b$ for some positive real r_b. Let $W_T = \prod_{b \notin T} r_b$, $F = \sum_T W_T \overline{T}$, where the summation is over all maximal trees. Then RF is self-adjoint. (This yields the result of Kirchhoff [Ann. Physik vol. 72 (1847), pp. 497-510], and for R the identity shows that the projection of C on its 1-cycle subspace is $(1/N)\sum_T \overline{T}$, where N is the number of maximal trees of G.) Theorem: if $b_1,\ldots,b_\mu$ are all the branches not in a maximal tree T, $\det(R\overline{T}b_i, \overline{T}b_j) = \sum_T W_T$, where (,) denotes inner product. (For R the identity, this proves that $\det(\overline{T}b_i, \overline{T}b_j) = N$.) Further, dual results for coboundaries are given. (This work supported under contract AF 33 (616)-2797.) (Received December 9, 1957.)

542-82. Frank Kozin: On probability averages of functions.

Let $[x(t);\ 0 \leqq t \leqq 1]$, $x(0) = 0$, with probability one, be a process with stationary independent increments whose characteristic function is exp (th(u)). If $\exp[-(\lambda/N)\sum_{k=1}^{N}[x(k/N)]^2]$ converges in probability to $\exp[-\lambda\int_0^1[x(t)]^2dt]$, $\lambda > 0$, then $E\,[\exp[-\lambda\int_0^1[x(t)]^2dt]\,] = \int_W\{\exp[\int_0^1 h(2\lambda^{1/2}x(t))dt]\}$, where $\int_W$ denotes the Wiener average. The proof makes use of the Parsival theorem for Fourier transforms. No moment considerations are necessary. This result is shown to be a special case of a slightly more general theorem for processes with stationary independent increments. There appears to be a connection between this theorem for the Cauchy process and a result of M. Kac [Trans. Amer. Math. Soc. vol. 59 (1946) p. 401]. (Received December 9, 1957.)

542-83. Martin Schechter: On the coerciveness of linear second order partial differential operators.

Let A be a linear second order partial differential operator with complex coefficients. The coefficients are assumed continuous in the closure $\overline{G}$ of a bounded domain G in n-space. Let E be the set of all complex functions which are twice continuously differentiable in $\overline{G}$ and whose first derivatives are real on the boundary. If D^2 denotes the generic second derivative, there is a constant K depending only on G and A such that $\|D^2u\|_{L_2(G)} \leqq K(\|Au\|_{L_2(G)} + \|u\|_{L_2(G)})$ for all $u \in E$ if and only if A is elliptic and there does not exist a continuous function γ in $\overline{G}$ such that the leading coefficients of γA are real. The same result holds if E is taken to be the set of real functions which are twice continuously differentiable in $\overline{G}$. (Received December 9, 1957.)

542-84. J. J. McKibben: Singular integrals in two dimensions.

Let $q(x_1,x_2)$ be a polynomial in two real variables x_1 and x_2. It is shown that, if $u(x_1, x_2)$ is a C^∞ function which is rapidly decreasing at ∞, it is possible to define, in the sense of Hadamard, the finite part of the singular integral $\int_{R^2}\{u(x_1,x_2)/q(x_1,x_2)\}dx_1dx_2$. In the language of Laurent Schwartz's distributions, this amounts to a proof of the existence of pseudo-functions of the type $1/q(x_1,x_2)$. As a corollary, it is shown that every partial differential operator $q(\partial/\partial x_1, \partial/\partial x_2)$ in two independent variables and with constant coefficients possesses a tempered elementary solution, i.e. a tempered distribution E satisfying the equation $q(\partial/\partial x_1, \partial/\partial x_2)E = \delta$, where δ is the Dirac delta function. The method employed in the proof is an adaptation of Hadamard's method of approximating R^2 by a sequence of regions in which the function $1/q$ is regular. (Received December 10, 1957.)

542-85. C. C. Hsiung: *Curvature and Betti numbers of compact Kaehler manifolds with boundary*.

The purpose of this paper is to study compact Kaehler manifolds with boundary. The author first extends Hodge's theorem concerning the relation between the effective tensor fields and Betti numbers of compact Kaehler manifolds without boundary, and then uses this result and a previous one [C.C. Hsiung, *Curvature and Betti numbers of compact Riemannian manifolds with boundary*, Bull. Amer. Math. Soc. Abstract 63-6-778] to extend Bochner's theorems on compact Kaehler manifolds without boundary. The simplest result in the second part can be stated as follows. If a compact Kaehler manifold of real dimension $2n$ with a connected boundary B of real dimension $2n - 1$ has positive constant holomorphic curvature, then the absolute and relative Betti numbers of the manifold are given by the equations $B_{2j} = B_{2j} \pmod{B} = 1$, $B_{2j+1} = B_{2j+1} \pmod{B} = 0$, $(0 \leqq 2j \leqq n - 1)$. (Received December 10, 1957.)

542-86. G. L. Krabbe: *Normal operators on a Banach space*.

Let $\mathfrak{X}$ be a reflexive Banach space with adjoint $\mathfrak{X}'$. Let $S(\mathfrak{X})$ be the set of all bounded operators on $\mathfrak{X}$ that have an extension Q to $\mathfrak{X} \cup \mathfrak{X}'$ such that $\langle Qx,y\rangle = \langle x,Qy\rangle$ for all (x,y) in $\mathfrak{X}\times\mathfrak{X}'$. The set $N(\mathfrak{X})$ of normal operators is here defined as the set of all operators $T = T_1 + iT_2$, where $(T_1,T_2) \in S(\mathfrak{X}) \times S(\mathfrak{X}')$; it is moreover required that the residual spectrum of T be void and that there exists a nondecreasing, projection-valued function E defined on the spectrum of T, while the points of discontinuity of E coincide with the point-spectrum of T. This extends the notion of normal operators on a Hilbert space. This article supplies examples of families of such operators in case $X = \ell_p (1 < p < \infty)$; the families are subrings of the algebra (H_p) of convolution operators studied in [Hartman, Proc. Amer. Math. Soc. vol. 18 (1957) pp. 45-48]. Two function-rings are found to correspond to two sub-rings (H_p^0) and (H_p^1) of (H_p); it turns out that $(H_p^0) \subset (H_p^1) \subset N(\mathfrak{X})$. If $T \in (H_p^0)$, then $T = \int \lambda \cdot dE(\lambda)$ holds in the weak topology of $\mathfrak{X}$. All members of (H_p^0) are spectral operators in the sense of Lorch, but in general, the members of (H_p^1) do not satisfy the conditions of Dunford and Lorch (although they are "normal" in the sense defined above). (Received December 10, 1957.)

542-87. W. S. Loud: *Periodic solutions of a perturbed autonomous system*.

In (1) $x' = g(x) + \epsilon f(t,x,\epsilon)$ ($' = d/dt$) let x be a real n-vector, and let $g(x)$ and $f(t,x,\epsilon)$ be in C^2 for appropriate values of (t,x,ϵ). Let $x' = g(x)$ have a periodic solution $x_0(t)$ of least period L_0, and let $f(t,x,\epsilon)$ be periodic in t of period T, a rational multiple of L_0. Let L be the l.c.m. of T and L_0. Let the variation equation (2) $y' = g'(x_0(t))y$ have no solution of period L linearly independent of $x_0'(t)$. Levinson (Ann. Math 52 (1950) pp. 727-738) does not assume T/L_0 rational. In the present paper existence and stability of solutions of (1) are investigated using a technique of Coddington and Levinson (*Theory of ordinary differential equations*, Chapter 14). Let $z_0(t)$ be the unique solution of period L of the system adjoint to (2) with $\int_0^L z_0^*(t)x_0'(t)dt = 1$. It is shown that (1) has a solution of period L of the form $x = x_0 + \epsilon x_1 + o(\epsilon)$ for small ϵ if $z_0(t)$ is orthogonal to $f(t,x_0(t),0)$ on $[0,L]$ and not orthogonal to $f_t(t,x_0(t),0)$. If (2) has $(n - 1)$ characteristic multipliers of magnitude $\neq 1$, then the variation equation $y' = (g'(x) + \epsilon f_x(t,x,\epsilon))y$ has for small $\epsilon \neq 0$ one more characteristic multiplier of magnitude less than 1 (greater than 1) than has (2) if $\epsilon\int_0^L z_0^*(t)f_t(t,x_0(t),0)dt$ is positive (negative). This work was supported in part by the Office of Ordnance Research. (Received December 10, 1957.)

542-88. R. H. Cameron and M. D. Donsker: *Inversion formulae for characteristic functionals of stochastic processes*.

Let $\{x_t, 0 \leqq t \leqq 1\}$ be a stochastic process almost all of whose sample functions are in L_2 and vanish at the origin. We define the characteristic functional of the process to be $\Phi(p) = E\{\exp[i\int_0^1 x(t)dp(t)]\}$ where $\{p_t, 0 \leqq t \leqq 1\}$ is the Wiener process with almost all its sample functions, $p(t)$, vanishing at the origin. For processes $\{x_t, 0 \leqq t \leqq 1\}$ almost all of whose sample functions satisfy a Holder condition of order $\alpha > 1/2$, the following inversion formula is obtained for functions $G[x]$ which are bounded and almost everywhere continuous in the uniform topology: Let $0 < f(\lambda) = o(\lambda/(\log\lambda)^2)$ but $1/f(\lambda) = o(\lambda^{1-1/\alpha})$. Then $E_x\{G[x]\} = \lim_{\lambda\to\infty} 2^{-1/2}\exp(\lambda f/2)\, E_a^w E_p^w\{\exp[-i\lambda f\int_0^1 a(t)dp(t)]G[fa]\Phi(\lambda p)\}$. Extensions are made to other Gaussian inversion measures and other regularity assumptions. (Received December 10, 1957.)

542-89. Anil Nerode: *Linear automaton transformations*.

Let R be a finite commutative ring with unit, let N consist of the non-negative integers, let R^N be the set of all functions on N to R. A map M on R^N to R^N

is a linear automaton transformation (LAT) if: M is linear under pointwise operations; there exists a finite set Q, a $\bar{q}$ in Q, maps $M_Q: R \times Q \to Q$, $M_R: R \times Q \to R$ such that for f in R^N there exists an h in Q^N with $h(0) = \bar{q}$, $h(n + 1) = M_Q(f(n), h(n))$, $(Mf)(0) = 0$, $(Mf)(n + 1) = M_R(f(n), h(n))$. Theorem: $M: R^N \to R^N$ is a LAT if and only if there exists a matrix $W_{nk}: N \times N \to R$ such that: for f in R^N, $n > 0$, $(Mf)(n) = W_{n0}f(0) + \dots + W_{n(n-1)}f(n - 1)$; $u_{ij} = W_{(i+j)j}$, $i \geqq 1$, $j \geqq 0$, has an eventually periodic sequence of rows and an eventually periodic sequence of columns. Another necessary and sufficient condition is that there exists an equation $b(n) + u_1(n - 1)b(n - 1) + \dots + u_p(n - p)b(n - p) = t_1(n - 1)a(n - 1) + \dots + t_p(n - p)a(n - p)$ (where $u_1, \dots, u_p, t_1, \dots, t_p$, a, b map $\{0, \pm 1, \pm 2, \dots\}$ into R and vanish for negative arguments) with $u_1, \dots, u_p, t_1, \dots, t_p$ eventually periodic and such that; for f in R^N there exist a, b satisfying the equation with $a(n) = f(n)$, $b(n) = (Mf)(n)$ for $n \geqq 0$. (Received December 10, 1957.)

542-90. V. E. Beneš: On exponential holding-time II. Continuous time treatment for a trunk group serving a renewal process.

A generating function is obtained for the transition probabilities of $n(t - 0)$, the number of trunks (out of N) busy at t, defined so as to be continuous from the left. It is assumed only that the trunk holding-times are negative exponential, and that calls arrive in a renewal process. The results are obtained in two ways: first, by appeal to a continuous parameter Markov process in two dimensions, and second, by taking advantage of the regenerative properties of $n(t - 0)$. It is shown that the distribution of $n(t - 0)$ approaches a limit as t becomes large; this limit is the stationary distribution. Formulas for the mean and covariance of $n(t - 0)$ are given. (Received December 10, 1957.)

542-91. V. E. Beneš: On exponential holding-time IV. The character of overflow traffic.

A finite group of trunks with exponential holding-times is considered. The group serves calls arriving in a renewal process. The stochastic process consisting of calls which overflow this group (by finding all trunks busy) is completely described. In particular, it is shown that this process is again a renewal process, and a general formula is given for the Laplace-Stieltjes transform of the distribution of interoverflow times. Some applications and examples are included. (Received December 10, 1957.)

542-92. Vera Pless: Quotient rings of continuous transformations rings.

Let M and N be dual vector spaces and let L(M,N) denote the ring of all continuous transformations on M and F(M,N) the ring of finite valued transformations in L(M,N). An element a in L(M,N) has property A (B) if its range is closed and the dimension of its kernel (codim of range) is finite. Let H be the set of elements in L(M,N) which have both properties A and B and whose adjoints have properties A and B. Then H is the inverse image, under the natural map, of the group of regular elements in L(M,N)/F(M,N). If we define the index of a, written i(a), for a in H, to be the codimension of the range of a minus the dimension of its kernel, then i(a + f) = i(a) for f in F(M,N) and i(ab) = i(a) + i(b). The index function thus gives rise to a homomorphism between the group of regular elements in L(M,N)/F(M,N) and the additive group of integers. Similar results were proven by Desprit (Annales de la Société Scientifiques de Bruxelles 22, March 1957, serie 1) for N the full dual of M. If M is a Banach space and N is its continuous dual, it is then shown, using a result of Yood (Pacific J. Math. vol. 4 (1954) pp. 615-636), that L(M,N)/F(M,N) has a radical. (Received December 10, 1957.)

542-93. Vera Pless: On the continuous transformation ring of biorthogonal bases spaces.

Dual vector spaces M and N over a division ring D are called biorthogonal bases spaces if there exist equipotent bases $\{x_i\}$ for M and $\{y_j\}$ for N such that $(x_i, f_j) = \delta_{ij}$. Let L(M,N) denote the ring of all continuous transformations on M and let F(M,N) denote the ring of finite valued transformations in L(M,N). Let L(M) denote the ring of all linear transformations on M and F(M) the ring of all finite valued transformations in L(M). It is known (R. E. Johnson, Equivalence rings, Duke Math. J. vol. 16 (1948) p. 789) that any two-sided ideal I of L(M) consists of all those linear transformations of rank less than some infinite cardinal $\aleph_I$ where $\aleph_I \leq$ dim M. From this it can be proven (I. Kaplansky, Notes on the theory of rings, University of Chicago, 1950, p. 18) that L(M)/F(M) is primitive without one-sided minimal ideals, further that the quotient of two two-sided ideals in L(M) has this structure. In this paper it is shown that exactly the same results hold if L(M,N) is substituted for L(M) and F(M,N) is substituted for F(M). Finally it is proven that the quotient of any two ideals in L(M,N) cannot be isomorphic to the quotient of two ideals in L(M). (Received December 10, 1957.)

542-94. Shreeram Abhyankar: Fundamental groups of algebraic manifolds.

For brevity π_1 stands for its tower of subgroups of finite index. Theorem. Let V be a nonsingular r-dimensional complete algebraic variety (arbitrary ground field). Let $W_1, \ldots, W_t$ be irreducible (r-1)-dimensional subvarieties of V and $W = W_1 + \ldots + W_t$. Assume: (1) the dimension of the complete linear system determined by each W_i is greater than 1, (2) W has only normal crossings, (3) V is simply connected; then $\pi_1(V-W)$ is generated by t generators. Assume further: (4) the W_i are pairwise connected; then $\pi_1(V-W)$ is abelian. Remark. Omitting (3) gives the effect on π_1 of removing W. Corollary 1. Let V be projective space ($r > 1$), keep (2) and observe that (1), (3), (4) are automatic. Hence $\pi_1(V-W)$ is abelian with t generators. Let $g_i =$ degree of W_i. It is proved that the t generators $a_1, \ldots, a_t$ can be so chosen that $a_1^{g_1} \ldots a_t^{g_t} = 1$ is the only relation. For $r = 2$ and the complex ground field this is Zariski's theorem. Corollary 2. Consider V^*: $x_{r+1}^g - f(x_1, \ldots, x_r) = 0$ where $f = 0$ is any irreducible hypersurface of degree g with only normal crossings (e.g. generic) in r-space V. Projecting V^* onto V and applying Corollary 1 to V-W we conclude that V^* is simply connected. For $r = 2$ and the complex ground field this was asserted in Picard-Simmart. (Received December 11, 1957.)

542-95. Eugene Schenkman: The similarity between the properties of ideals in commutative rings and the properties of normal subgroups of groups. Preliminary report.

The radical of a normal subgroup A of a group G is defined to be the group union of all normal subgroups K so that K/A is solvable. A normal subgroup equal to its radical is called a radical. A normal subgroup A is irreducible if G/A is not a direct product. An irreducible radical P has the property that if the commutator [A,B] of two normal subgroups is in P then either A or B is in P. The main result in this connection is that a radical A (such that G/A has a finite principal series) has a unique minimal decomposition as intersection of irreducible radicals -- the "primes" of A. The residual quotient A:B of two normal subgroups A and B is defined to be the maximal group Q such that [Q,B] is in A. A consequence of the above result is that if A and B are radicals A:B = A if and only if no prime of B is contained in a prime of A. Other analogies to ideal theory are noted. (Received December 11, 1957.)

542-96. M. S. Robertson: Schlicht Dirichlet series.

For a function F(s) defined by a Dirichlet series $F(s) = \sum a_n e^{-\lambda_n s}$, $a_1 = -1$, $s = \sigma + it$, whose abscissa of absolute convergence is $\bar{\sigma}$, $-\infty \leqq \bar{\sigma} \leqq \infty$, there is a smallest number τ which we shall call its class, $\bar{\sigma} \leqq \tau < \infty$, for which $\sum_2^\infty \lambda_n |a_n| e^{-\lambda_n \tau} \leqq 1$. Although F(s) is not schlicht for $RS > \tau$, it is shown that all the functions F(s) of the same class τ are uniformly locally schlicht, in the sense of P. Montel, for $RS > \tau$ when $\lambda_1 \tau < \log \lambda_1$ and for $RS > \tau_1$ when $\lambda_1 \tau \geqq \log \lambda_1$ where $\tau_1(\lambda_q - \lambda_1) = \lambda_q \tau - \log \lambda_1$, and q is the suffix of the first nonvanishing coefficient a_q of the numbers a_n, $n \geqq 2$. The Theorem is a best possible one. More precisely it is shown that for every ϵ, $0 < \epsilon < 1$, F(s) is schlicht in every circle $|S - S_0| \leqq (1 - \epsilon)\pi/\lambda_1$ for $RS_0 > \alpha$ where α is found as an explicit function depending only upon τ, λ_1, λ_q and ϵ. The factor π/λ_1 in the radius cannot be replaced by a larger one. Sufficient conditions are found to insure that F(s) is schlicht in semi-infinite strips. For example, if F(s) is of class τ, $\lambda_1 \tau < \log \lambda_1$, then F(s) is schlicht in every semi-infinite strip of width $2t_0$ which is parallel to the real axis and lies in the half-plane $RS \geqq \tau$, t_0 being defined by $\lambda_1 \cos(\lambda_1 t_0) = \exp \lambda_1 \tau$, $0 < 2\lambda_1 t_0 < \pi$. (Received December 11, 1957.)

542-97. J. S. MacNerney: Generalized chain sequences. Preliminary report.

Suppose $\{S,Q\}$ is a complete inner-product space, i. e., S is a vector space over the (real or complex) numbers and Q is an inner-product for S and S is complete with respect to the norm induced by Q. Let B be the set of continuous linear mappings in $\{S,Q\}$, A^* the adjoint of A for A in B, and the statement that $H \gg 0$ mean that H is in B and $H^* = H$ and $Q(x,Hx) \geqq 0$ for x in S. If $H \gg 0$ then $H^{1/2}$ denotes the T such that $T \gg 0$ and $T^2 = H$, $H^{-1/2}$ denotes the inverse of the restriction of $H^{1/2}$ to the closure in S,Q of H(S), and the statement that $K = A^* H^{-1} A$ means that A is in B and A(S) is a subset of $H^{1/2}(S)$ and $H^{-1/2}A$ is in B and $K = (H^{-1/2}A)^*(H^{-1/2}A)$. Theorem I. If $H \gg 0$ and $K \gg 0$ and A is in B the following are equivalent: (1) $|Q(x,Ay)|^2 \leqq Q(x,Hx)Q(y,Ky)$ for x and y in S, (2) $K - A^* H^{-1} A \gg 0$. Theorem II. If each of $\{a_i\}_1^\infty$ and $\{b_i\}_1^\infty$ is a sequence in B and n is a positive integer the following are equivalent: (1) if $\{x_i\}$ is a sequence in S then $\sum_{i=1}^{n+1} Q(x_i, a_i x_i) - 2 \operatorname{Re} \sum_1^n Q(x_i, b_i x_{i+1}) \geqq 0$, (2) there is a sequence $\{M_{ni}\}_{i=0}^n$ in B such that $a_{n+1} = M_{nn} \gg 0$ and if $1 \leqq i \leqq n$ then

$a_i - b_i M_{ni}^{-1} b_i^* = M_{n,i-1} >> 0$. *Theorem* III. If II (2) holds for all n then, for $k \geqq 1$, the sequence $\{M_{n,k-1}\}_{n=k}^{\infty}$ converges (strongly!) to a limit M_{k-1} and $M_{k-1} = a_k - b_k M_k^{-1} b_k^*$. (Received December 11, 1957.)

542-98. P. C. Hammer: *Functions of bounded variation* I.

There is a need for a greater variety of measures of approximation and a greater variety of functional representations in all phases of applied mathematics. In this paper fundamental aspects of concepts associated with the total variation of real-valued functions on simply ordered sets are considered including finite estimates of the total variation, additivity of the variation functional, and generalizations of the variation concept to include the representation of increasing continuous functions as a composition of concave and convex increasing functions. It is proved that a necessary and sufficient condition that the variation of the sum of two functions be the sum of their variations is that the functions be syntonic (in a sense defined). (Received December 11, 1957.)

542-99. W. J. Hardell: *Left associated elements in a linear algebra*. Preliminary report.

Let A be a linear associative algebra with an identity and a finite basis of order k over a field F of characteristic 0. Let M(k,F) be the set of all k-rowed square matrices over F. Using well known structure theorems we select a special basis for A. The correspondence S(a) ⟷ a, where S(a) is the left regular representation of a with respect to the special basis, defines an isomorphism between A and a subset of M(k,F). Two elements a and b in A are left associates if there exists a regular element s such that sa = b. The main theorem is: a necessary and sufficient condition that a and b are left associates is that their corresponding left regular representations S(a) and S(b) have the same Hermite normal form in M(k,F). The necessity is trivial. For the sufficiency we first show if US(a) = S(b) for some U in M(k,F) we can construct an element u in A such that S(u)S(a) = S(b). In general if U is unimodular, S(u) is not. The crux of the proof is to show how to construct regular v and p such that the element w formed from S(v)US(p) is regular. (Received December 11, 1957).

542-100. Alfred Lehman: *A remark on wye-delta transformations*.

In this abstract a network theory is a set of bilateral network configurations together with a set of branch values closed under network composition. If a two-terminal network is constructed, it must be equivalent across its terminal-pair to some branch whose branch value will then be the driving-point value of the network. A network theory is weakly reciprocal if there exists a group operation on the set of branch values such that given a two-terminal-pair network, the driving-point value across pair one plus (group operation) the driving-point value across pair two with pair one short-circuited equals the value measured across pair two plus that measured across pair one with pair two short-circuited. A balance condition is required in order that a weakly reciprocal network theory admit wye-delta and delta-wye transformations. This condition is expressed by the two-terminal equivalence of the two networks $(a \cup b \cup c) \cap ((a \cap b) \cup (a \cap c) \cup (b \cap c))$ and $(a \cap b \cap c) \cup ((a \cup b) \cap (a \cup c) \cap (b \cup c))$ where $\cup$ and $\cap$ denote series and parallel composition of two-terminal branches. The balance condition, a simple example of abstract method, proves useful in application. (Received December 11, 1957.)

542-101. J. H. B. Kemperman: On the asymptotic distribution of a sequence in a compact group.

Let G_1, G_2 denote compact Hausdorff spaces, let $x_n \in G_1$, $y_n \in G_2$, $(n = 1,2,..)$. If $\lim_N \sum_a^{a+N} f(x_n)/N = \mu \cdot f$, for each real continuous function f on G_1, the Borel measure μ (assumed regular) is called the distribution of $\{x_n\}$. In the following, G_1 = compact group, written additively. Consider: $(^*)\{x_n + hy_n\}$ has a uniform distribution (= Haar measure) on G_1, h denoting a map $G_2 \rightarrow G_1$. For instance, $(^*)$ holds if $\{x_n\}$ has a uniform distribution on G_1, $\{x_n\}$, $\{y_n\}$ are independent, (i.e. $\{(x_n, y_n)\}$ in $G_1 \times G_2$ has a product measure as its distribution), and h is continuous a.e. $[\nu]$, (ν = distribution of $\{y_n\}$). This generalizes a result due to L. E. Grosh, (Thesis, Purdue, 1954). Let also G_2 = compact group. For k a fixed integer $\geqq 0$, let $x'_n = \Delta_{s_1} \dots \Delta_{s_k} x_n$, similarly y'_n, (s_i = integer > 0, $\Delta_s z_n = z_{n+s} - z_n$). Then $(^*)$ holds for each continuous map h whenever, for each choice of the s_i, $\lim y'_n = 0$ and $\{x'_n\}$ has a uniform distribution on G_1, such that the above limit holds uniformly in a for each f, (e.g. when G is abelian, $\Delta_1 x'_n \rightarrow x$, $\{nx\}$ dense in G). The same conclusion holds if, for each choice of the s_i, $\{x'_n\}$ has a uniform distribution on G_1 and $\{x'_n\}$, $\{y'_n\}$ are independent. In the latter case, $\{x_n\}$, $\{y_n\}$ are even independent as soon as $\{y_n\}$ possesses a distribution. (Received December 11, 1957.)

542-102. V. L. Shapiro: The curl of a vector field.

Operating in three dimensional Euclidean space and using the notation $X = (x,y,z)$ and $(X,X_0) = (xx_0 + yy_0 + zz_0)$. Let $V(X) = [V^1(X), V^2(X), V^3(X)]$ be a continuous vector field defined in a neighborhood of the point X_0. Define curl_1 of V at X_0 to be the vector $\text{curl}_1 V(X_0) = [F^1(X_0), F^2(X_0), F^3(X_0)]$ provided $F^j(X_0)$ exist and are finite $(j = 1,2,3)$ where $F^j(X_0) = \lim_{t \to 0} (\pi t^2)^{-1} \int_{C_j(X_0,t)} (V,dX)$ and $C_1(X_0,t)$, $C_2(X_0,t)$, $C_3(X_0,t)$ are respectively the circumference of the circles with center X_0 and radius t and normals parallel respectively to the x,y, and z axes. The following theorem then holds: Let D be a bounded domain in Euclidean three space, and let V(X) be a continuous vector field defined in D. Suppose (i) curl_1 V is defined and continuous in D, (ii) S is an oriented regular surface in D whose boundary C is a closed regular curve directed in accordance with the given orientation on S, (iii) S + C is in D. Then Stokes Theorem holds for S, i. e. $\int_C (V,dX) = \int_S (\text{curl}_1 V,n)dS$. The main idea in the proof is to use the uniqueness theory of multiple trigonometric series. A similar theorem can be established for the curl defined by means of the cross product over the surface of spheres. (Received December 11, 1957.)

542-103. W. M. Boothby and H. C. Wang: Homogeneous contact manifolds.

A manifold M of dimension 2n + 1 is said to be a contact manifold if there is defined over it a Pfaffian form ω such that the exterior product $\omega(d\omega)^n \neq 0$ anywhere on M. If $\lambda \neq 0$ is a function on M, then $\lambda\omega$ defines an equivalent contact structure. If we restrict ourselves to compact M, the only classical examples known to the authors are the unit tangent bundle of an arbitrary n + 1 dimensional compact manifold, and the spheres S^{2n+1} (which are not of the former type). The form ω determines a nonvanishing vector field on M. If the integral curves of this field are closed then M is a circle bundle over a symplectic manifold whose fundamental 2-form Ω determines an integral cohomology class, the characteristic class of the bundle, and a contact form equivalent to ω defines a connection in the bundle. Due to a theorem of Kobayashi the converse also holds. In the case that a compact Lie group acts transitively on M the integral curves are shown to be closed and M is a circle bundle over a homogeneous Kähler space. Not all the examples thus obtained are unit tangent bundles over n + 1 dimensional manifolds although some of them are. (Received December 11, 1957.)

542-104. C. J. Neugebauer: *A decomposition of a Peano space.*

In a previous abstract (Bull. Amer. Math. Soc. Abstract 63-2-340) a B-set B of a Peano space P is defined as a nondegenerate continuum B of P such that either $B = P$ or else every component of $P - B$ has a finite frontier. A fine-cyclic element Δ of P (introduced first in a slightly different context by L. Cesari, Bull. Amer. Math. Soc. Abstract 61-6-698) is a B-set which remains connected after removing any finite set. B-sets and fine-cyclic elements are extensions of A-sets and proper cyclic elements. The following theorem gives an interesting relation between fine-cyclic elements and proper cyclic elements. *Theorem*. If the degree of multicoherence r(P) of P is finite, then there exists a finite number of B-sets $B_1, \ldots, B_n$ of P such that (1) $P = B_1 \cup \ldots \cup B_n$; (2) $B_i \cap B_j$ is either empty or else finite, $i \neq j$ $(i,j = 1,\ldots,n)$; (3) each fine-cyclic element of P is a proper cyclic element of a unique B_i $(1 \leqq i \leqq n)$; (4) each proper cyclic element of B_i is a fine-cyclic element of P. If $r(P) = \infty$, the theorem does not hold in general. (Received December 11, 1957.)

542-105. C. J. Neugebauer: *Local A-sets and retractions onto B-sets.*

A nondegenerate closed subset B of a Peano space P will be termed a *local A-set* of P provided either $B = P$ or else there exists a connected open set G of P containing B such that every component of $G - B$ has a single frontier point relative to G, and if θ', θ'' are two components of $G - B$ with distinct frontier points (relative to G) then their closures (relative to P) are disjoint. It is shown that every local A-set is a B-set (see above abstract) and, in case the underlying Peano space is of finite degree of multicoherence, the converse is also true. If B is a local A-set of P, then there is a natural retraction t from G (a connected open set as above) onto B. The mapping t is the identity on B and sends every component of $G - B$ into its frontier relative to G. Such a retraction can be extended continuously to P so as to map P onto B and $P - B$ into a dendrite (possibly degenerate). If B' is a local A-set of B and B is a local A-set of P, then for any retraction t from P onto B, there exists a retraction t' from B onto B' such that t't is a retraction from P onto B'. (Received December 11, 1957.)

542-106. C. J. Neugebauer: *A fine-cyclic additivity theorem for a functional*.

Let $\mathfrak{T}$ be the class of all continuous mappings (T,A) from any open subset A of a Peano space P into a metric space P^*. Two mappings (T',A'), (T'',A'') in $\mathfrak{T}$ constitute a *partition* of (T,F) in $\mathfrak{T}$ provided there is a finite set $F \subset P^*$ and a pair of closed sets $E' \neq \phi$, $E'' \neq \phi$ such that $(\alpha) P = E' \cup E''$, $E' \subset A'$, $E'' \subset A''$; $(\beta) T'(x) = T(x), x \in E'$, and $T'(A' - E') \subset F$; $(\gamma) T''(x) = T(x)$, $x \in E''$, and $T''(A'' - E'') \subset F$; $(\delta) T(E' \cap E'') \subset F$. Let Φ be a real-valued nonnegative functional defined on $\mathfrak{T}$ such that (1) if $(T_n, F) \rightarrow (T,F)$ uniformly on P, $\Phi(T,P) \leqq \liminf \Phi(T_n,P)$; (2) if (T',A'), (T'',A'') constitute a partition of (T,P), then $\Phi(T,F) = \Phi(T',A') + \Phi(T'',A'')$; (3) $\Phi(T,A) = 0$ whenever (T,A) admits an unrestricted factorization (T,A) = sf, $f: A \rightarrow M$, $s: M \rightarrow P^*$ with M a dendrite. For $(T,P) \in \mathfrak{T}$, let (T,P) = sf, $f: P \rightarrow M$, $s: M \rightarrow P^*$ be an unrestricted factorization where M is a Peano space of finite degree of multicoherence. For Δ a fine-cyclic element of M, let G_Δ be a connected open set of M containing Δ so that (see above abstract) Δ is a local A-set (relative to G_Δ). Let t_Δ be the natural retraction from G_Δ onto Δ and let $A_\Delta = f^{-1}(G_\Delta)$. *Theorem*. $\Phi(T,P) = \sum \Phi(st_\Delta f, A_\Delta)$, $\Delta \subset M$. This formula extends to fine-cyclic elements a cyclic additivity theorem due to E. J. Mickle and T. Radó (Trans. Amer. Math. Soc. vol. 66 (1949) pp. 347-365). (Received December 11, 1957.)

542-107. L. Cesari and C. J. Neugebauer: *On the coincidence of Geöcze and Lebesgue areas*.

Let (T,A) be a continuous mapping from a closed finitely connected Jordan region $A \subset E_2$ into E_3, and let L(T,A), V(T,A) be the Lebesgue and Geöcze areas of (T,A). The proof of L(T,A) = V(T,A) (see L. Cesari, *Surface area*, Princeton, 1956, where a direct proof of this equality is given) includes: (*) If $L(T,A) < \infty$, then L(T,A) = V(T,A). In case A is a 2-cell another and earlier proof of (*) is due to T. Radó (*Length and area*, Amer. Math. Soc. Colloquium Publications, vol. 30, 1948). This paper extends Radó's proof to the case where A is a Jordan region of connectivity $\geqq 1$. Let $(T,A) = \ell m$, $m: A \rightarrow M$, $\ell: M \rightarrow E_3$ be a monotone-light factorization of (T,A) and let $\{\Delta\}$ be the collection of fine-cyclic elements of M (L. Cesari, Abstract 61-6-698; C. J. Neugebauer, Abstract 63-2-340 this Bulletin). With each Δ there can be associated a fine-cyclic mapping (T_Δ, J_Δ) where J_Δ is a finitely connected Jordan region in A (a mapping (T,A) is fine-cyclic provided M is a fine-cyclic element). As a corollary of a representation theorem of W. Fleming (Abstract 542-114.) it follows that, if $L(T_\Delta, J_\Delta) < \infty$, then $L(T_\Delta, J_\Delta) = V(T_\Delta, J_\Delta)$. Results of L. Cesari (Abstract 61-6-698) and C. J. Neugebauer (Abstract 542-106) yield fine-cyclic additivity formulas $L(T,A) = \sum L(T_\Delta, J_\Delta)$, $V(T,A) = \sum V(T_\Delta, J_\Delta)$, $\Delta \subset M$. Thus if $L(T,A) < \infty$, then $L(T,A) = V(T,A)$. (Received December 11, 1957.)

542-108. D. L. Boyer and E. A. Walker: Almost locally pure Abelian groups. Preliminary report.

Let an Abelian group G be called almost locally pure (hereafter abbreviated a. l. p.) if for every finite set of elements $g_1, \ldots, g_n$ of G there exists a finitely generated pure subgroup of G which contains $g_1, \ldots, g_n$. Some of the properties of a. l. p. groups are the following: a p-group is a. l. p. if and only if it has no elements of infinite height; a countable a. l. p. group is a direct sum of cyclic groups; every subgroup of an a. l. p. group is an a. l. p. group; it is known what homomorphic images with pure kernels of a. l. p. groups are a. l. p.; in particular, if T is the torsion subgroup of an a. l. p. group, then G/T is a. l. p. (Received December 11, 1957.)

542-109. E. A. Walker: Subdirect sums and infinite Abelian groups. Preliminary report.

Define a group to be a rational group if it is a subgroup of a $Z(p^\infty)$ group or a subgroup of the additive group of rational numbers. From the fact that every Abelian group can be imbedded in a divisible group, it is proved that every Abelian group G can be represented as a weak subdirect sum of rational groups in such a way that G intersects each subdirect summand nontrivially. Using this theorem, the following two theorems are proved: (1) Every torsion free Abelian group G of infinite rank has, for every possible infinite index α, $2^{o(G)}$ pure subgroups of order equal to o(G) and of index α. Furthermore, the intersection of these pure subgroups of index α is o. (2) Every torsion free Abelian group G of infinite rank can be represented as a weak subdirect sum of copies of the additive group of rational numbers, and in such a way that G intersects each subdirect summand nontrivially. (Received December 11, 1957.)

542-110. R. B. Crouch: Characteristic subgroups of monomial groups.

Let U be a set, H a group, B an infinite cardinal, and B^+ the successor of B. Let o(u) = B where o(u) means the number of elements of U. The set of monomial substitutions on the elements of U with coefficients in H forms a group $\sum(H, B, B^+, B^+)$. The set $\sum(H, B, C, D)$ of substitutions of the form $y = vs$ where v is a multiplication with less than C nonidentity factors, s is a permutation that permutes less than D elements of U, is a subgroup. The normal subgroups of $\sum(H, B, \aleph_0, D)$, $\aleph_0 \leqq D \leqq B^+$ are known (R. B. Crouch, Trans. Amer. Math. Soc. vol. 80 (1955) pp. 187-215 and R. B. Crouch and W. R. Scott, Proc. Amer. Math. Soc. vol. 8 (1957) pp. 931-936). The automorphisms of these groups are known (Calvin V. Holmes, *Contributions to theory of groups*, Research Grant N S F - G 1126, Report No. 5, pp. 23-93). This paper combines the results of this information to determine the characteristic subgroups of $\sum(H, B, \aleph_0, D)$ and various other subgroups of $\sum(H, B, B^+, B^+)$. (Received December 11, 1957.)

542-111. C. J. A. Halberg, Jr., E. Levin and E. G. Straus: *On contiguous congruent sets in Euclidean space*.

Two plane sets are *contiguous* if their intersection is nonempty and is equal to the intersection of their frontiers. A set A *encloses* a set B if every unbounded connected set which intersects B also intersects A. The following theorem holds. *Let* S *be the closure of any bounded region in* E^2. *Then for any direction* θ *there exist six translates of* S *with the following properties*: (1) *one translate is in the direction* θ. (2) *Each translate is contiguous to* S. (3) *No two translates have interior points in common*. (4) *Each translate is contiguous to two others*. (5) *The union of the six translates encloses* S. A more restrictive concept is the following. The sets A, B in E^n are *adjacent* if they are on opposite sides of a common plane of support and have limit points in common (adjacency and contiguousness are equivalent for closed convex bodies). Let k(n) be the maximal number of nonintersecting open unit spheres in E^n which can be adjacent to a given unit sphere. Then the following theorem holds. *Let* S *be a bounded set in* E^n, *then there exist sets* $S_1,\dots,S_{k(n)}$ *in* E^n *such that*: (1) *Every* S_i *is the reflection of* S *on a hyperplane*. (2) *Every* S_i *is adjacent to* S. (3) *The convex hulls of* S_i *and* S_j *have no interior points in common for* $i \neq j$. (Received December 11, 1957.)

542-112. C. J. A. Halberg, Jr. and V. A. Kramer: *Concerning a theorem of Gelfand and Levitan*. Preliminary report.

Let $\mu_1 \leqq \mu_2 \leqq \mu_3 \leqq \dots$ be the eigenvalues of the differential operator $-(d/dx)^2$ with boundary conditions $y'(0) - hy(0) = 0$, $y'(\pi) + Hy(\pi) = 0$.

Let $\lambda_1 \leqq \lambda_2 \leqq \lambda_3 \leqq \ldots$ be the corresponding eigenvalues of the operator $-(d/dx)^2 + q(x)$ with the same boundary conditions. Suppose that $q(x)$ is differentiable and that $\int_0^\pi q(x)dx = 0$. The problem of evaluating the sum $\sum_1^\infty (\lambda_i - \mu_i)$ was first considered by Gelfand and Levitan [Akademiia Nauk, S. S. S. R. Doklady vol. 88 (1953) pp. 593-596] but the formula given by them is not correct since $q(x) \equiv 0$, $hH \neq 0$ yields a counterexample. The present paper uses different methods to attack this problem. First, the sum in question is proved equal to the sum of the first order perturbation terms. Second, this sum is evaluated by means of Sturm-Liouville expansion theorems. More general problems of the same nature are considered. (Received December 11, 1957.)

542-113. A. H. Stroud: Functions of bounded variation. II.

Let f be any bounded, real-valued function defined on a simply ordered set. Define $V_k(f)$ to be the supremum of finite sums of the form $\sum |f(x_i) - f(x_{i-1})|$ for no more than k elements x_i for which $x_{i-1} < x_i$, $(k \geqq 2)$. The following conjecture of Hammer is proved: $V_{k+2}(f) - V_k(f)$ is nonincreasing with respect to k. By reducing the problem to the case where there always exists elements x_i for which the supremum is attained the proof is carried out by considering the relationship between these elements and the other relative extrema of the order. As a consequence of this theorem, if g is a function such that $V_{m+2}(g) - V_m(g) = 0$ for some m, then $V_m(g)$ is the total variation of g. (Received December 11, 1957.)

542-114. W. H. Fleming: Nondegenerate surfaces and fine-cyclic surfaces.

The notion of fine-cyclic decomposition is due to L. Cesari and has proved a fruitful tool for extending surface area theory from the 2-cell case to higher topological types. Let S stand for a Fréchet surface in N-space, of the type of a finitely connected Jordan region, and L(S) its Lebesgue area. This paper proves the existence of a favorable representation for any fine-cyclic [Cesari-Neugebauer, Abstract No. 542-107 above] surface S with L(S) finite. Theorem. Any such surface S has a representation (T,J), where J is a compact planar region bounded by a finite number of circles, with the following properties: (1) T is continuous on J; (2) L(S) is given by the classical integral formula; and (3) there exists a compact set $K \subset J$, which is the union of finitely many analytic arcs, such that J-K is connected and the restriction of T to any closed Jordan region $J_1 \subset J - K$ is a Dirichlet mapping. It can happen that S has no Dirichlet representation, in contrast to the situation for nondegenerate surfaces

[Morrey, Amer. J. Math. vol. 57 (1935) pp. 692-702, vol. 58 (1936) pp. 313-322; Fleming, Bull. Amer. Math. Soc. Abstract 63-1-98]. (Received December 11, 1957.)

542-115. R. L. Wilson: A prediction problem in real-time.

Let f(t) be a continuous function in real time for which no information is available other than the (N + 1) values $f(t_0 - nT)$, $n = 0, 1, 2, \ldots, N$. A function $p(t_0 + kT)$, $k > 0$, is derived which approximates the value of $f(t_0 + kT)$. This function is a linear combination of the known values of f(t) with coefficients dependent only on N and k. An explicit expression is obtained for the error $f(t_0 + kT) - p(t_0 + kT)$ in the case where k is an integer. (Received December 12, 1957.)

542-116. J. H. Case: Some snake-like continua. Preliminary report.

A continuum is hereditarily equivalent provided that it is nondegenerate and is topologically equivalent to each of its nondegenerate subcontinua. By means of a characterization (due to R. H. Bing) of hereditarily decomposable, snake-like continua; it is shown that any decomposable, hereditarily equivalent continuum is snake-like. Lida Barrett observed that the same proof gives the theorem—any hereditarily decomposable, hereditarily irreducible continuum is snake-like. It is shown that any continuum which is an inverse limit of arcs is snake-like. In view of the fact that the geometric realization of the nerve of a chain covering is an arc, one might expect that any snake-like continuum is the inverse limit of the geometric realizations of the nerves of its open coverings. The pseudo-arc is a snake-like continuum which does not have this property. (Received December 12, 1957.)

542-117. R. W. Bagley and J. D. McKnight, Jr.: On Wallman's method.

The method of H. Wallman for constructing a compactification of a T_1 space is applied to other embedding problems by using collections of closed sets with the countable intersection property (c.i.p.). If X is a space such that every collection of closed sets with the c.i.p. is contained in a collection of closed sets maximal with respect to the c.i.p., then X is densely embeddable in a Lindelöf space X^* obtained by Wallman's method from the collections of closed sets maximal with respect to the c.i.p. X^* is Hausdorff if X is normal, and X^* is compact if X is countably compact. A somewhat different application is obtained for arbitrary Tychonoff spaces X. If Wallman's method is modified so as to use only those collections of zero sets (of continuous functions) with the c.i.p. which are maximal among collections of zero sets with the finite intersection property, a Q-space [Hewitt 1948] containing X as a dense subset is obtained. (Received December 12, 1957.)

542-118. E. F. Beckenbach: Subfunctions and inequalities.

Continuous subadditive functions and generalized convex functions are characterized in terms of boundedness on a set S such that the interior measure of $S + S$ is positive; further, the role of these functions in the establishment of algebraic and function-theoretic inequalities is discussed. (Received December 12, 1957.)

542-119. S. G. Bourne: On ideal theory in a commutative semigroup.

In this present work, the decomposition of s-ideals into the intersection of primary ideals is studied, assuming only the ascending chain condition for ideals, thus paralleling the "algebraic-geometrical" aspect of ideal theory in a ring, in which ideals are used to study algebraic manifolds. The main results are: Any ideal I of S is the intersection of primary ideals of S. The decomposition $I = [Q_1, Q_2, \dots, Q_m]$ into primary ideals is said to be an irredundant decomposition if $[Q_1, Q_2, \dots, Q_{j-1}, Q_{j+1}, \dots, Q_m] \supset I$, $j = 1, 2, \dots, m$. If $I = [Q_1, Q_2, \dots, Q_m]$ is an irredundant decomposition into primary ideals of S and the corresponding radicals, $P_1, P_2, \dots, P_m$ are distinct, then this decomposition is said to be short. If $I = [Q_1, Q_2, \dots, Q_m] = [Q'_1, Q'_2, \dots, Q'_{m'}]$ are two short decompositions of I into primary ideals of S, then $m = m'$ and the corresponding radicals $P_1, P_2, \dots, P_m$ coincide in some order with the corresponding radicals $P'_1, P'_2, \dots, P'_{m'}$. If $I = [Q_1, Q_2, \dots, Q_m] = [Q'_1, Q'_2, \dots, Q'_m]$ are two short decompositions of I and if $I_1 = [Q_{i_1}, Q_{i_2}, \dots, Q_{i_m}]$ and $I = [Q_{i_1}, Q_{i_2}, \dots, Q_{i_m}]$ are corresponding isolated components, then $I_1 = I'_1$. (Received December 12, 1957.)

542-120. Albert Edrei and W. H. J. Fuchs: A gap theorem for entire functions with deficient values.

Let $f(z) = \sum_{v=0}^{\infty} f_v / z^v$ be a nonconstant entire function of finite order λ and let Δ denote the sum of the deficiencies of all the deficient values of f(z). There exists a positive function $\psi(\epsilon, \lambda)$ depending on λ and ϵ (> 0) (but not on the function f(z)), such that $\Delta > 2 - \psi(\epsilon, \lambda)$ has the following consequences: (I)

$\lambda + 1/2$ is not an integer; (II) if p is the integer closest to λ, then $(S/p) - \epsilon \leqq D \leqq \overline{D} \leqq S/p$, where S is one of the integers $1, 2, \ldots, p$ and D, $\overline{D}$ denote, respectively, the lower and upper densities of the sequence $\{f_v\}$. (Received December 12, 1957.)

542-121. H. P. Kramer: Best finite linear approximation to a second order stochastic process.

Let x_t be a stochastic process with $Ex_t = 0$ and $r_{st} = Ex_s\overline{x}_t < \infty$, T an interval of real numbers such that $r_{st} \in L_2(T \times T)$ and N a fixed positive integer. Denote by S_N the set of all linear expressions of the form $y_t = \sum_{k=1}^{N} r_k f_k(t)$ composed of N linearly independent $f_k \in L_2(T)$ and $r_k \in L_2(x)$. Here $L_2(x)$ signifies the L_2 completion of the linear hull of x_t. Moreover, let $\Phi_k(t)$ and λ_k be defined as the solutions of the equation $\lambda F(t) = \int_T r_{ts} F(s)\, ds$ with $\lambda_1 > \lambda_2 > \ldots$. Under the above conditions and with the described notation, an optimal approximation to x_t by $y_t \in S_N$ is given by the Theorem: $\min_{y_t \in S_N} E \int_T |x_t - y_t|^2 dt = \sum_{k=N+1}^{\infty} \lambda_k$ is achieved by $y_t = \sum_{k=1}^{N} \lambda_k^{1/2} v_k \Phi_k(t)$ with $\lambda_k^{1/2} v_k = \int_T x_t \overline{\Phi_k(t)} dt$. (Received December 12, 1957.)

542-122. Pentti Laasonen: A Ritz method for simultaneous determination of several eigenvalues and eigenvectors of a matrix.

Let A be an $(n \times n)$-matrix, Y and V two arbitrary $(n \times m)$-matrices $(m \leqq n)$. Let $Y^{(k)} = A^k Y$ define the iterated matrices $Y^{(k)}$, and $Q^{(k)} = V'Y^{(k)}$ the $(m \times m)$ kernel matrices $Q^{(k)}$. If the roots of the equation $\det(A - \lambda I) = 0$ are $\lambda_i (i = 1, 2, \ldots, n)$ and m of them are larger in absolute value than the remaining $n - m$, then in general, that is for arbitrarily chosen Y and V, the roots of $\det(Q^{(k+1)} - \mu Q^{(k)}) = 0$ tend to the m roots λ_i in question, as $k \longrightarrow \infty$; the corresponding nontrivial solutions $x_i^{(k)}$ of $(Q^{(k+1)} - \mu_i^{(k)} Q^{(k)})x = 0$, multiplied on the left by $Y^{(k)}$, tend simultaneously to the associated eigenvectors of A. If A is hermitian and V has been chosen $V = \overline{Y}$, and consequently $Q^{(k)}$: s are also hermitian, then this method gives in general, in the above sense, without restrictions on $m \leqq n$, m' eigenvalues and eigenvectors, where m' is the largest number not exceeding m such that m' eigenvalues of A are in absolute value greater than the remaining $n - m'$. The method is useful if from a large matrix only a small number of eigenvalues and eigenvectors are to be determined. (Received December 12, 1957.)

542-123. E. A. Michael: Yet another characterization of paracompact spaces.

If $\mathcal{U}$ and $\mathcal{V}$ are collections of subsets of a topological space X, call $\mathcal{V}$ closed in $\mathcal{U}$ if there exists a function assigning to each $V \in \mathcal{V}$ a $U_V \in \mathcal{U}$ such that, for every $\mathcal{V}' \subset \mathcal{V}$, $(\cup\{V \mid V \in \mathcal{V}'\})^- \subset \cup\{\bar{U}_V \mid V \in \mathcal{V}'\}$, where the bars denote closure. Examples: (1) $\mathcal{V}$ is a closure preserving (particularly, a locally finite) refinement of $\mathcal{U}$; (2) $\mathcal{V}$ is an open star-refinement (in Tukey's sense) of $\mathcal{U}$. The following result generalizes Theorems 1 and 2 in [Proc. Amer. Math. Soc. vol. 8 (1957) pp. 822-828]: Theorem: The following properties of a regular space X are equivalent: (a) X is paracompact; (b) Every open covering $\mathcal{U}$ of X has a (not necessarily open) refinement which is closed in $\mathcal{U}$; (c) Every open covering $\mathcal{U}$ of X has an open refinement $\mathcal{V} = \cup_{i=1}^{\infty} \mathcal{V}_i$, with each $\mathcal{V}_i$ closed in $\mathcal{U}$. Note that A. H. Stone's theorem asserting that fully normal spaces are paracompact follows directly from the examples and (b). (Received December 12, 1957.)

542-124. James Radlow: Diffraction by a dielectric wedge. Preliminary report.

The problem of the diffraction by a (right-angled) dielectric wedge is that of finding a function $u(x_1,x_2)$ which may be written as the sum of a given incident wave and an unknown scattered wave in all of space, with the scattered wave $\hat{u}(x_1,x_2)$ required to satisfy $(\Delta_{x_1x_2} + \lambda^2)u = 0$, for $x_1 \geqq 0$, $x_2 \geqq 0$ $(\Delta_{x_1x_2} + \mu^2)u = 0$, for $E[x_1 < 0 \cup x_2 < 0]$ where $\Delta_{x_1x_2}$ is the two-dimensional Laplacian and λ, μ are distinct complex constants. It is further required that the total field u and its normal derivative be continuous across the boundary of the wedge (i.e., across the positive x_1- and x_2- axes), and that the scattered field $\hat{u}$ be outgoing at infinity. The problem is shown to be equivalent to that of solving a certain equation involving four unknown functions of two complex variables, and this equation is treated by a function-theoretic process which generalizes the classical technique of Wiener-Hopf to two complex variables. The underlying ideas of the generalization have been given in the author's earlier paper, Diffraction by a quarter-plane, Comm. Pure Appl. Math. (to appear). In the present case, the analysis is mainly concerned with the construction of a required two-variable factorization. The final result is an explicit integral representation, valid in all of space, for the scattered field $\hat{u}(x_1,x_2)$. (Received December 12, 1957.)

542-125. F. A. Raymond: *Cech homology from a chain complex.*

Let X be locally compact and U an open subset of X. We define a complex K. A p-chain ϕ^p will be a function on all p + 1 ordered compact sets of X into a group G such that (1) $\bigcap_{i=0}^{n} A_i = \emptyset \Rightarrow \phi^p(A_0,\ldots,A_p) = 0$, (2) $A_i = A_i' \cup A_i''$ and $\text{Int } A_i' \cap \text{Int } A_i'' = \emptyset \Rightarrow \phi^p(A_0,\ldots,A_i,\ldots,A_p) = \phi^p(A_0,\ldots,A_i,\ldots,A_p) + \phi^p(A_0,\ldots,A_i'',\ldots,A_p)$. We define supports for the chains and show that K is a carapace in the sense of Leray-Cartan. Let K_c denote the subcarapace of chains with compact support. K_c possess a boundary operator and when X is a countable union of compact sets K_c is fine. The homology of the chains with compact support mod the chains whose support (compact) is in U is isomorphic to the Cech Homology group, H(X,U) [compact carriers and field coefficients]. Kolmogoroff originally gave a modified version of the above chains. Lefschetz showed in a still different version that one could obtain an isomorphism with the Cech homology groups of a normal space (using finite covers of closed sets). Our proof is a modification of Lefschetz's argument to account for our above definition. (Received December 12, 1957.)

THE APRIL MEETING IN CHICAGO, ILLINOIS

April 18-19, 1958

544-1. Eben Matlis: Injective modules.

Let R be a ring with identity. The injective envelope of an R-module M is denoted by E(M). E(M) is indecomposable if and only if M has no nonzero submodules S,T such that $S \cap T = 0$. If I is a finite intersection of irreducible left ideals, then E(R/I) is a finite direct sum of indecomposable injectives. If R is left-Noetherian, then every injective R-module is a direct sum of indecomposable injectives. If R has the minimum condition on left ideals, then every injective is determined up to isomorphism by a finite collection of cardinal numbers. Henceforth R will be a commutative, Noetherian ring. There is a 1-1 correspondence between the prime ideals of R and the indecomposable injectives given by $P \leftrightarrow E(R/P)$. An injective module is determined up to isomorphism by assigning a cardinal number to each prime ideal of R. If P is a prime ideal of R, then $\mathrm{Hom}_R(E(R/P),E(R/P))$ is naturally isomorphic to C, the completion of the local ring derived with respect to P. E(R/P) is the union of an ascending sequence of finitely-generated C-submodules A_i. The A_{i+1}/A_i are finite-dimensional vector spaces over the residue field of C, and their dimensions are related to the symbolic prime powers of P. (Received December 4, 1957.)

544-2. W. R. Wasow: Solution of nonlinear differential equations with a parameter by asymptotic series.

Consider the differential equation $\epsilon x = f(t,x,\epsilon)$ for the n-dimensional vector function $x(t,\epsilon)$ of the complex variable t and the positive parameter ϵ. Denote by $w_0(t)$ a solution of $f(t,w_0(t),0) = 0$ and suppose that f is holomorphic in all variables in a neighborhood of $t = 0$, $x = w_0(t)$, $\epsilon = 0$. Assume that the eigenvalues $\lambda_j(t)$ of the matrix of partial derivatives of $f(t,w_0(t),0)$ with respect to the components of $w_0(t)$ are distinct and different from zero in a neighborhood of $t = 0$. Then the differential equation possesses for $|t| \leqq t_0$ a solution $x = x_0(t,\epsilon)$ that admits, as $\epsilon \rightarrow +0$, an asymptotic series in powers of ϵ. If, in addition, all $\lambda_j(t)$ lie on the same side of a line through $t = 0$, and if $\sum_{k=1}^{n} \mu_k \lambda_k(t) - \lambda_k(t) \neq 0$ for all $j = 1,2,\ldots,n$ and for all nonnegative integers μ_k such that $\sum_{k=1}^{n} \mu_k \geqq 2$, then there exists a sector with vertex at $t = 0$ where any solution $x(t,\epsilon)$ with initial values close to $w_0(0)$ can be developed into an absolutely convergent series of the form $x(t,\epsilon) = \sum_{r=0}^{\infty} x_r(t,\epsilon) \exp\{(1/\epsilon) q_r(t)\}$. The $q_r(t)$ are the functions $\sum_{k=1}^{n} \mu_k \int_0^t \lambda_k(s)ds$ nonnegative integers. The $x_r(t,\epsilon)$ possess asymptotic series in powers of ϵ, as $\epsilon \rightarrow +0$. The coefficients of these series can be calculated algebraically. (Received December 23, 1957.)

544-3. Frank Harary: *On the group of the composition of two graphs*.

A binary operation on linear graphs, called *composition*, is constructed with the property that the automorphism group of the composition of two graphs is in general permutationally equivalent to thecomposition (the "Gruppenkranz" of Pólya [Acta Math. vol. 68 (1937) pp. 145-254]) of their groups. The only exception occurs when both graphs are complete. The problem was suggested by the work of Frucht [Bull. Amer. Math. Soc. vol. 55 (1949) pp. 418-420] who gave the result for the case that the first graph in the composition is totally disconnected. Two other products of graphs are considered: the *cartesian product* studied by Sabidussi [Canadian J. Math. vol. 9 (1957) pp. 515-525] and the *join* of Zykov [Mat. Sbornik vol. 24 (1949) pp. 163-188]. Conditions are given for the permutational equivalence of (a) the group of the cartesian product of two graphs with the cartesian product [defined in F. Harary, *The number of k-colored graphs*, to appear in Pacific J. Math.] of their groups and (b) the group of the join of two graphs with the direct product of their groups. (Received January 23, 1958.)

544-4. A. H. Wallace: *Analytic equivalence of curves*.

Let V and V' be algebroid curves with ideals $\mathfrak{a}$ and $\mathfrak{a}'$ in $k[[x_1,x_2,\ldots,x_n]]$. V,V' are defined to be analytically equivalent if there is an automorphism T of $k[[x_1,x_2,\ldots,x_n]]$ such that $T(\mathfrak{a}) = \mathfrak{a}'$. *Theorem*. *Let* $\mathfrak{a} = (F_1,F_2,\ldots,F_{n-1})$, $\mathfrak{a}' = (F'_1,F'_2,\ldots,F'_{n-1})$ *and let* $W \subset V$ *have components corresponding to the isolated primes of* $\mathfrak{a}$. *Then if the orders of the series* $F_i - F'_i$ *are high enough*, W *is analytically equivalent to a subvariety of* V'. The proof is by induction on n, the case $n = 2$ having been proved by Samuel (J. Math. pures et appl. vol. 35 (1956) pp. 1-6). The induction hypothesis is applied to the projections of V,V' on $x_n = 0$, the resulting analytic equivalence being lifted into n-space by a topological argument in which the family of all algebroid curves is suitably topologized. (Received February 6, 1958.)

544-5. D. R. Hughes: On the H_p-subgroup of a group. Preliminary report.

Let G be a group and p a prime, and define $H_p(G)$ as the subgroup of G generated by the elements which do not have order p. We say that $H_p(G)$ is trivial if $H_p(G) = 1$ or G. It is easy to show that for any group G, $H_2(G)$ is trivial or $[G: H_2(G)] = 2$. Here we obtain the following: (1) if G is finite and not a p-group, then $H_p(G)$ is trivial or $[G: H_p(G)] = p$; (2) if $H_p(G)$ is not trivial and is abelian, and if $[G: H_p(G)]$ is finite, then $[G: H_p(G)] = p$. As a consequence of (2), one finds: (3) for any group G, $H_3(G)$ is trivial or $[G: H_3(G)] = 3$; the results of the Burnside problem for 3 are necessary to prove (3) in the infinite case.

(Received February 14, 1958.)

544-6. A. S. Householder: On minimal norms.

Associated with every finite, bounded, convex body K containing the origin in its interior there is a unique vector norm such that $x \in K$ if and only if $\|x\|_K \leqq 1$. For any matrix A define AK by $y \in AK$ if $y = Ax$ with $x \in K$. Let α be the least non-negative scalar satisfying $AK \subset \alpha K$. Then $\alpha = \|A\|_K$ is the norm of A with respect to K. It is known that $\|A\|_K \geqq \rho(A)$ for any K, where $\rho(A)$ is the spectral radius of A. If equality holds, the norm will be said to be minimal for A, and K to be minimizing for A. If K is minimizing for both A and B, then $\rho(AB) \leqq \rho(A)\rho(B)$, and, for any non-negative scalars α and β, $\rho(\alpha A + \beta B) \leqq \alpha\rho(A) + \beta\rho(B)$. For any K define [K] by $y \in [K]$ if $y = \omega x$ where $|\omega| \leqq 1$ and $x \in K$, and [K] will be said to be the result of equilibrating K. If K is minimizing for A, then also [K] is minimizing for A. The complete class of matrices is characterized for which each of the following convex bodies is minimizing after equilibration: an ellipsoid; a polytope with vertices on the coordinate axes; a parallelotope with faces parallel to the coordinate hyperplanes.

(Received February 17, 1958.)

544-7. P. T. Church: On cluster sets in n-dimensional space.

Let D^n be the open unit ball in E^n. The cluster set C(f,p) of a function f: $D^n \rightarrow E^n$ at a point p on Bdy(D^n) is the set of all points y (say on the n-sphere, S^n, regarded as the compactification of E^n) such that there exists a sequence of points $x_m \rightarrow p, x_m$ in D^n, with $f(x_m) \rightarrow y$. If f is continuous, C(f,p) (relative to S^n) is a continuum. W. Gross proved (Monatshefte für Mathematik und Physik vol. 29 (1918) pp. 20-21) that given any closed, connected set C in the plane and at a point p on Bdy(D^2), there is a meromorphic function f: $D^2 \rightarrow E^2$ where C(f,p)

$= C$. Using different techniques the author has proved: (1) Given any continuum C in S^n ($n \geqq 2$), there is an infinitely differentiable, quasi-conformal local homeomorphism f: $D^n \rightarrow E^n$ such that $C(f,p) = C$. (2) f may be chosen to be a homeomorphism if and only if $S^n - C$ has a component whose frontier is precisely C. (f quasi-conformal means: there exists a real number $B > 1$ such that, given any point q in the domain of f, and the directional derivatives d_1 and d_2 in any two directions at q, then $d_1/d_2 < B$.) (Received February 18, 1957.)

544-8. W. L. Hart and T. S. Motzkin: *Proof of the fundamental theorem on implicit functions by composite gradient corrections*.

Consider the system $*$: $f(x,t) = 0$, where $x = (x_1, \ldots, x_n)$, $t = (t_1, \ldots, t_k)$, $f = (f_1, \ldots, f_n)$ are real, $\partial f_j / \partial x_i$ continuous in a neighborhood Ω of $(\alpha;\beta)$, $f(\alpha;\beta) = 0$, and the matrix $A = (a_{ij})$, $a_{ij} = \partial f_j(\alpha;\beta)/\partial x_i$, is nonsingular. Let $w_j = (\sum_i a_{ij}^2)^{1/2}$ and ψ_{ij} be the ith direction angle of the positive gradient of $f_j(x;\beta)$ at $x = \alpha$. Define $\{x^k\} = \{x^k(t)\}$ by $x^0 = \alpha$ and $x^k = x^{k-1} + \Delta x^{k-1}$, $k > 0$, with the vector $\Delta x^{k-1} = \sum_j \Delta_j x^{k-1}$, a composite gradient correction (Hart and Motzkin, Pacific J. Math. vol 6), where $\Delta_j x_i^{k-1} = -\rho f_j(x^{k-1};t) \cdot w_j^{-1} \cos \psi_{ij}$, $\rho > 0$. Thus $x^k = \Phi(x^{k-1};t)$, $k > 0$, where $\Phi = (\phi_1, \ldots, \phi_n)$, $\phi_i(x;t) = x_i - \rho \sum_j a_{ij} w_j^{-2} f_j(x;t)$, and $x = \Phi(x;t)$ is equivalent to $*$. For $\rho < 2/n$ there exist $\epsilon, \delta > 0$, $\delta \leqq \epsilon$, such that the region $(\|x - \alpha\| \leqq \epsilon, \|t - \beta\| \leqq \delta)$ is in Ω, and $\{x^k\}$ converges uniformly for $\|t - \beta\| \leqq \delta$ to a unique solution $x(t)$ of $*$ in $\|x - \alpha\| \leqq \epsilon$, with $x(t)$ continuous. The x^k are suitable for high-speed computation and involve A itself, instead of A^{-1}, used in the constructive proof of the same theorem by Goursat (Bliss, Princeton Colloquium Lectures, p. 16). (Received February 20, 1958.)

544-9. R. E. Langer: *Formal solutions and a related equation for a class of fourth order differential equations of a hydrodynamic type*.

The differential equation considered is (1) $d^4w/dz^4 + \lambda^2\{P_1(z,\lambda)d^2w/dz^2 + P_2(z,\lambda)dw/dz + P_3(z,\lambda)w\} = 0$, with λ a large parameter and $P_j(z,\lambda)$, $j = 1,2,3$, analytic in z and $1/\lambda$. The z-domain includes a *turning-point* z_0, where $P_1(z_0,\infty) = 0$. It is shown there are formal solutions which are power series in $1/\lambda$ with coefficients analytic in z, and that three formal solutions have the forms $w_j(z,\lambda) = A(z,\lambda)v_j(x,\lambda) + \lambda^{-1} B(z,\lambda)dv_j/dx + \lambda^{-2}C(z,\lambda)d^2v_j/dx^2$, $j = 1,2,3$, with x a certain function $x(z)$, and $d^3v/dx^3 + \lambda^2 x dv/dx - \lambda^2 \mu v = 0$. In general the solutions so obtained constitute a complete set. When they do not, a perturbation method is used to obtain an additional one. It is shown that appropriate truncations of the formal solutions give functions that solve a differential equation which differs arbitrarily little from (1). (Received February 20, 1958.)

544-10. G. S. Young: *Generalizations of the Liouville theorem to higher dimensions*.

The author proves the following results: *Theorem* A. Let D be a domain in E^n with compact closure. Let $f:\bar{D} \rightarrow E^n$ be a vector function, of class C' and with a non-negative Jacobian $J(f)$ in D. Suppose that R is a spherical region in D, of radius r, that f is one-to-one in R, and that in R, $J(f) \geqq k > 0$. Then the oscillation of f on $\bar{D} - D$ is not less than $2k^{-n}$. A corollary is *Theorem* B. Let $f:E^n \rightarrow E^n$ be a class C' and have a non-negative Jacobian. If f has a limit at infinity, then $J(f) \equiv 0$. Theorem B reduces to the classical Liouville theorem for analytic functions in the plane. Methods of proof are topological. (Received February 21, 1958.)

544-11. J. L. Bailey: *Thermal shock on the surface of the elastic half-space*.

The temperature distribution $T=$constant within a circle and zero outside is suddenly imposed and maintained on the plane surface of an elastic half-space initially at zero temperature. Under classical assumptions plus the additional assumption of small time, the stress solution, including consideration of the stress wave emitted, is obtained. (This result together with the quasi-static and steady-state solutions is contained in the author's doctoral thesis, *A thermoelastic problem in the half-space*, written under the supervision of Dr. H. Parkus.) After solution of the temperature problem, the corresponding Navier equations are transformed into the Laplace subsidiary space where a particular solution is produced by means of a thermoelastic potential. By use of a theorem from the theory of Laplace transformation (cf. Doetsch, *Handbuch der Laplace Transformation*, vol. 2, p. 45), the stresses in the original space derived from this potential are shown to satisfy the boundary conditions for small values of time. (Received February 26, 1958.)

544-12. R. S. Varga: *Orderings of the successive overrelaxation method*. Let $B = \| b_{i,j} \|$ be a non-negative symmetric $n \times n$ matrix with zero diagonal entries, and assume that B is irreducible (indecomposable) and that the largest eigenvalue in modulus of B, $\bar{\mu}(B)$, satisfies $0 < \bar{\mu}(B) < 1$. Assume, moreover, that B satisfies property (A) of D. Young [Trans. Amer. Math. Soc. vol. 76 (1954)]. For any permutation, or ordering, ϕ of the integers $1 \leqq i \leqq n$, let B_ϕ denote the matrix corresponding to the similarity transformation of B. If $B_\phi = L_\phi + U_\phi$, where L_ϕ, and U_ϕ are strictly lower and upper triangular matrices, then $\mathcal{L}_{\phi,\omega} \equiv (I - \omega L_\phi)^{-1}\{\omega U_\phi + (1 - \omega)I\}$ defines the successive overrelaxation operator with overrelaxation factor ω. For certain orderings ϕ^*, called consistent orderings, Young showed that the overrelaxation factor, ω_b, giving the fastest convergence for $\mathcal{L}_{\phi^*,\omega}$ satisfies: $\omega_b = 2/(1 + (1 - \bar{\mu}^2(B))^{1/2})$ where $1 < \omega_b < 2$. Theorem 1. If ϕ^* is any consistent ordering, then for all ordering ϕ and for any ω with $0 < \omega \leqq 2$, $\bar{\mu}(\mathcal{L}_{\phi,\omega}) \geqq \bar{\mu}(\mathcal{L}_{\phi^*,\omega})$. For the special case $\omega = 1$, this proves a conjecture of D. Young. Corollary. $(\bar{\mu}(B))^2 \leqq \bar{\mu}(\mathcal{L}_{\phi,1}) < \bar{\mu}(B)$ for all orderings ϕ. Theorem 2. If $R_{1,\phi} = -\ln \bar{\mu}(\mathcal{L}_{\phi,1})$, and $R_2 = -\ln \bar{\mu}(B)$, then $\lim_{\bar{\mu}(B)\uparrow 1} R_{1,\phi}/2R_2 = 1$ for all orderings ϕ. As a special case of the above theorem, we have a proof of a conjecture of Shortley and Weller [Journal of Applied Physics, vol. 9]. (Received February 24, 1958.)

544-13. L. A. Kokoris: On nilstable algebras.

A simple commutative power-associative algebra A of degree 2 over a field F of characteristic not 2 has a unity element $1 = u + v$ where u, v are orthogonal idempotents. Then A can be decomposed relative to u and written as $A = A_u(1) + A_u(1/2) + A_u(0)$ where x is in $A_u(\lambda)$ if and only if $xu = \lambda x$. It is known that $A_u(1)$ and $A_u(0)$ are orthogonal subalgebras, $A_u(1/2)^2 \subseteqq A_u(1) + A_u(0)$ and $A_u(1/2)A_u(\lambda) \subseteqq A_u(1/2) + A_u(1 - \lambda)$ for $\lambda = 0,1$. It is further known that $A_u(1) = uF + G_1$, $A_u(0) = vF + G_0$ where G_1, G_0 are nilalgebras. We call u a nilstable idempotent and A nilstable with respect to u if $A_u(1/2)A_u(\lambda) \subseteqq A_u(1/2) + G_{1-\lambda}$ for $\lambda = 0,1$. Thus every stable idempotent is nilstable. The main result is that A is a Jordan algebra if and only if A is nilstable with respect to two idempotents u, f such that $u \neq 1$, $f \neq 1$, $u + f \neq 1$, and such that f is not of the form $f = u + w_{12} + w_1 + w_0$ or $f = v + w_{12} + w_1 + w_0$ with w_{12} in $A_u(1/2)$, w_1 in G_1, w_0 in G_0. Examples are given to show that the restrictions on f are necessary. (Received February 27, 1958.)

544-14. Sibe Mardešić: Comparison of singular and Čech homology in locally connected spaces.

A natural homomorphism ν of singular homology (Čech cohomology) theory into the Čech homology (singular cohomology) theory is first defined in the category of all topological spaces. For paracompact spaces, which are lc_S^{p-1}, i.e. homologically locally connected up to dimension p - 1 and semi-p-lc_S, i.e. semi homologically locally connected in dimension p, the homomorphism ν is an isomorphism; local connectedness is taken in the sense of singular homology. From this result one derives, that for paracompact Hausdorff spaces the property lc_S^p implies the corresponding property based on Čech homology. Some examples concerning relations between various types of local connectedness are given, showing in particular that a space can be homologically locally 1-connected (singular homology) without being homotopically locally 1-connected. (Received February 27, 1958.)

544-15. R. B. Paine: Fixed points of the circle group acting on a cohomology n-sphere. Preliminary report.

P. A. Smith (cf. Lefschetz: Algebraic topology, Amer. Math. Soc. Colloquium Publications, vol. 27, 1942, pp. 350-373) has proved that the set of fixed points of a cyclic group of prime order acting on a compact cohomology n-sphere forms a k-sphere. A. Borel (Nouvelle demonstration d'un theoreme de P. A. Smith, Comment Math. Helv. vol. 29 (1955) pp. 27-39) has given another proof. His method is extendible to the proof of the following theorem: Let S denote the circle group acting on a compact cohomology manifold X of finite dimension and with Alexander-Cech cohomology of an n-sphere. Then the set F of fixed points forms a cohomology manifold with groups of an (n-k)-sphere, where k is even. There is no loss of generality in assuming S acts freely on X-F. There is then induced a principle fibre space p: $X-F \rightarrow Y-F$, where Y is the orbit space X/S. Associated with this fibre space is a spectral sequence (E_r) with $E_2^{p,q} = H^p(B, H^q(X-F))$, where B is a classifying space of S, and with E_∞ an appropriately graded group associated with H(Y-F). A consideration of this sequence after the manner of Borel concludes the proof of the theorem. (Received February 27, 1958.)

544-16. Oswald Wyler: *Damped free vibrations with a finite number of oscillations*.

Damped free vibrations are solutions of a differential equation (D): $\ddot{x} + f(x,\dot{x}) + g(x) = 0$, satisfying existence and unicity conditions for solutions, and the following conditions. (1) $xg(x) > 0$ for all $x = 0$. (2) $yf(x,y) \geqq 0$ for all x and y. (3) If $g(x) = G'(x)$, then $G(x)$ is unbounded for x tending to $+\infty$ or to $-\infty$. The geometry of solutions of (D) is studied under two additional assumptions. (4) There are no periodic, i.e. undamped solutions. (5) A solution $x(t)$ of (D) other than $x(t) = 0$ has only a finite number of zeros of $y(t) = \dot{x}(t)$. For such an equation, the critical point (0,0) is asymptotically stable. There are five types of equations satisfying these conditions, classified according to possible initial and ultimate signs of $y(t)$. An equation (D) is called symmetric if the family of its orbits in the (x,y)-plane is point-symmetric with respect to the origin (0,0). All symmetric equations belong to the same type, and it is shown how the nonsymmetric types can be obtained through the consideration of symmetric equations. (Received February 27, 1958.)

544-17. Henry de Beaumont and Johann Martinek: *Research into the sets of solutions of certain systems of nonlinear partial differential equations of higher order*.

Partial differential equations of the type $(X,Y)/(x,y) = E(X,Y)$ where $(X,Y)/(x,y)$ denotes the Jacobian $(\partial X/\partial x)(\partial Y/\partial y)-(\partial X/\partial y)(\partial Y/\partial x)$ and where $E(X,Y) \equiv F(x,y;\ X,Y;X_x m_y p,Y_x r_y s;\ldots)$ represents a differential operator, in which $X_x m_y p,Y_x r_y s$ denote partial derivatives or orders $(m + p)$, $(r + s)$ are considered in systems of equations in which Y may be expressed, in particular in terms of $Y = L(X)$ or vice versa, with L being a differential operator, or, more generally, $(X,Y)/(x,y) = E_1(X,Y)$, $(X,Y)/(x,y) = E_2(X,Y)$ or $(X,Y)/(x,y) = E_1(X,Y)$, $(X,U)/(x,y) = E_2(X,Y)$, $L(X) = U$. Considering sets of element pairs (X,Y) of solutions of such equations to be defined, we then place emphasis upon the case when given an element, $(X + \xi, Y + \eta)$ is also an element of a set of solutions of the same system of equations, with ξ,η being constants. Moreover, a one-to-one correspondence is assumed between X,Y within the elements (X,Y) of the related set. The following theorem is then given which either enhances the integration or establishes final incompatibility. *Theorem*. Let (X,Y) be an element pair solution of a system, in which X,Y are bi-uniquely related to each other, in which system one equation may be written in the form $(X,Y)/(x,y)$

$= E_1(X,Y)$. Then, except perhaps for solutions of the nonreducible form $X = f_1(x) + f_2(y)$, $Y = c_1 f_1(x) + c_2 f_2(y)$ with c_1, c_2 constants, there exists with a,b,c being three constants a linear relationship which reads $aX + bY + c = 0$. (Received February 28, 1958.)

544-18. Lee Lorch: On the Lebesgue constants for Laplace series.

It is shown here that the Lebesgue constants (norms) for Laplace series, L_n, $n = 1,2,\ldots$, equal $2(2/\pi)^{1/2} n^{1/2} + \int_0^1 \{ J_1(t) - (2/\pi)^{1/2} t^{-1/2} |\cos(t - 3\pi/4)| \} dt - 2^{1/2}(2/\pi) \sum_{m=1}^{\infty} (-1)^{[m/2]} m^{-1/2} (4m^2 - 1)^{-1} + o(1)$, as n becomes infinite, where $J_1(t)$ is the Bessel function of the first kind and order 1, and $[m/2]$ is the largest integer not exceeding $m/2$. T. H. Grenwall (Math. Ann. vol. 74 (1913); Trans. Amer. Math. Soc. vol. 15 (1914)) established $L_n = 2(2/\pi)^{1/2} n^{1/2} + o(n^{1/2})$. G. Szegö devised much simpler proofs of Grenwall's result (Proc. London Math. Soc. vol. 36 (1932); Math. Ann. vol. 108 (1933)). (Received February 28, 1958.)

544-19. G. R. Lehner: Extending homeomorphisms on the pseudo-arc.

Let $H_1, H_2, \ldots, H_n$ be disjoint subcontinua of the pseudo-arc M such that M is irreducible between each pair of them. Using some results of R. H. Bing (Duke Math. J. vol. 15 (1948) pp. 729-742) the author constructs chains of arbitrarily small mesh that cover M in a certain prescribed way relative to $H_1, H_2, \ldots, H_n$ and then proves the following. Suppose F_1 and F_2 are closed subsets of the pseudo-arc M and that each has exactly n components. Suppose T is a homeomorphism of F_1 onto F_2. Let m = maximum $(2, n - 1)$. Then a necessary and sufficient condition that T can be extended to a homeomorphism of M onto M is that for any m points $P_1, P_2, \ldots, P_m$ of F_1, there exists a homeomorphism of M onto M taking P_i onto $T(P_i)$ $(i = 1,2,\ldots,m)$. (Received February 28, 1958.)

544-20. Günter Lumer: Numerical range and Banach space geometry. Preliminary report.

A general notion of numerical range (extending that classical Hilbert space concept) is introduced for every operator on a Banach space. It is shown, using analytical function theory instead of the classical approach, that most of the properties carry over to the general situation. Results of H. F. Bohnenblust and S. Karlin (Geometrical properties of unit sphere in Banach algebras, Ann. of Math. vol. 62 (1955)) can be easily derived. Similar and stronger results are established. For example, the concept of point of uniform convexity is introduced (i.e. $x_n + y_n/2 \to x_0; \|x_n\|, \|y_n\|, \|x_0\| = 1 \to \|x_n - y_n\| \to \theta$). A point of u.c. of the unit sphere S of a Banach space, is an extreme point of S; and while it is not difficult to show (following an idea of V. L. Klee) that any Banach space can be (equivalently) renormed in such a way that a given point of S, becomes an extreme point of the new unit sphere but not a point of u.c., it can also be proved that every unitary element u of a Banach algebra with unit (i.e. $u \in S$, $u^{-1} \in S$) is always a point of u.c. of S. Again, applications to function-algebras (in general; and *algebras) are discussed. (Received February 28, 1958.)

544-21. WITHDRAWN.

544-22. J. H. Roberts: *Contractibility of spheres in spaces of homeomorphisms*.

Suppose that M is compact, metric, and of finite dimension $\leqq n$, I_k is the k-dimensional interval, H_k is the space of homeomorphisms and C_k is the space of mappings (continuous) of M into I_k, with the usual metric topology. The author has shown that if $k \geqq 2n + 2$ then every 0-sphere S^0 (a pair of points) in H_k is contractible in H_k. The present result is a generalization: If $k \geqq 2n + 2 + r$ then every r-sphere S^r in H_k is contractible in H_k. Furthermore, if S^r is in C_k, then S^r is contractible in the union of S^r and H_k, the topology being that induced by C_k. This last result, for the case $r = 0$, has the following consequence: If f_1 and f_2 are different mappings (in C_k) then they are the end points of an arc in C_k each of whose points except f_1 and f_2 is a homeomorphism. (Received March 3, 1958.)

544-23. Harry Pollard and Harold Widom: *Inversion of a class of convolutions*.

The convolution with kernel k, (*)$f(x) = \int_0^\infty k(y)\phi(x + y)\,dy$ $(x > 0)$, is inverted under the assumptions (i) $k \in L(0,\infty)$, (ii) $k \not\equiv 0$ in a neighborhood of 0, (iii) the Fourier transform $\hat{k}(z) = \int_0^\infty e^{izx} k(x)dx$ has no zeros in the closed upper half-plane, (iv) $\phi \in L_p(0,\infty)$ for some p in $1 \leqq p \leqq 2$. The formal inversion formula $\phi(x) = (2\pi)^{-1}\int_0^\infty f(x + y)dy \int_{-\infty}^\infty e^{ity}(\hat{k}(-t))^{-1}dt$ is shown to hold when suitably modified. THEOREM: *Under hypotheses* (i)-(iv) *the equation* (*) *is inverted by either of the following formulas:* $\phi(x)$ $= \operatorname{l.i.m.}^{(p)}_{\delta \to 0+} \lim_{\epsilon \to 0+} (2\pi)^{-1} \int_0^\infty f(x + y)dy \int_{-\infty}^\infty e^{it(y-\delta)-\epsilon|t|} (\hat{k}(-t))^{-1}dt$;
$\phi(x) = \lim_{\delta \to 0+} \lim_{\epsilon \to 0+} (2\pi)^{-1} \int_0^\infty f(x+y)dy \int_{-\infty}^\infty (i\delta t)^{-1}(1-e^{-i\delta t})e^{ity-\epsilon|t|} (\hat{k}(-t)^{-1})dt$

almost everywhere (and in particular at all points of right continuity of ϕ). (Received March 3, 1958.)

544-24. G. M. Petersen: Bounded sequences summed by matrices.

If B is a regular sequence to sequence transformation that sums all bounded A-summable sequences, we shall write $B \supseteqq A$. The following has been proved by Brudno. Theorem. Let the sequence of matrices, $\{A^k\}$ $A^k = (a^k_{mn})$ be such that $A^1 \subseteq A^2 \subseteq A^3 \subseteq \dots \subseteq A^k \subseteq \dots$ and suppose that the matrix A sums every bounded sequence summable by at least one A^k. Then A sums a bounded sequence not summable by A^k for every $k = 1,2,3,\dots$ or A is equivalent to A^k for bounded sequences for some k. A new proof for this theorem is presented, if $\{s^k_n\}$ is not A^k summable but is A^n-summable, $n > k$ to zero, then for certain α_k, the sequence $\{s'_n\}$, $s'_n = \sum_1^\infty \alpha_k s^k_n$ is A-summable but not A^k-summable for any k. If the sequence of matrices is finite, A can be taken to be A^k for the largest k. (Received March 3, 1958.)

544-25. Paul Erdös and G. G. Lorentz: On the probability of relatively prime values of a function.

Let f(x) be an increasing function, g(x) the integral part of f(x). If Q(x) is the number of $n \leqq x$ for which $(n,g(n)) = 1$, the probability that n and g(n) are relatively prime is the limit of $Q(x)/x$ for $x \to \infty$. One can expect that this probability exists and is equal to $6\pi^{-2}$ for most functions f(x). This is proved for smooth f(x) under the assumptions (A) $f(x) = o(x/\log_2 x)$ and (B) $xf'(x)/\log_3 f(x) \to \infty$. The last condition roughly means that f(x) increases more rapidly than $\log x \log_3 x$. (B) is the best possible limitation from below, while (A) cannot be relaxed to $f(x) = \mathcal{O}(x/\log_3 x)$. These results are connected with the easier problem of equidistribution of f(n) modulo one. (Received March 3, 1958.)

544-26. W. M. Kincaid: Convergence of a two-point method for the numerical solution of sets of simultaneous equations.

In an earlier report (Bull. Amer. Math. Soc. Abstract 63-6-700) the author described a family of iterative methods, referred to as two-point methods, for the solution of sets of simultaneous equations. The methods, which may be regarded as generalizations of the classical method of false position, converge to the solution in a fixed number of steps when the equations are linear, and it was conjectured that they display second-order convergence in other cases. Convergence can now be demonstrated, under appropriate conditions, for one method of the type considered, which appears especially stable in this regard. (Received March 3, 1958.)

544-27. E. L. Lima: Stable Postnikov invariants.

In the S-category (see Spanier and Whitehead, Proc. Nat. Acad. Sci. vol. 39 (1953) pp. 655-660) an object is essentially a sequence $[X_j]$ where X_j is the j-th suspension of a given space X. One enlarges this category by allowing as objects sequences $[X_j]$ where X_{j+1} is not exactly the suspension of X_j but only 2j-equivalent to this suspension in the S-theoretical sense. By doing so, one is able to realize arbitrary S-homotopy groups and define for each space X (or, more generally, for any object) a system of invariants, similar to those of Postnikov, but with the added property of being stable under suspension. These invariants completely characterize the stable homotopy type of a CW-complex. (Received March 3, 1958.)

544-28. M. A. Martino, Jr. and G. J. Habetler: On the multigroup diffusion model.

The multigroup diffusion equations of reactor kinetics can be defined as follows: Let R be a simply-connected finite region in E_3 with $R = \bigcup_{i=1}^{m} R_i$, R_i convex, disjoint. Let n be a positive integer and let there be given real-valued functions $d_j(r)$, $a_{jk}(r)$, $(1 \leqq j,k \leqq n)$ bounded and C^2 in the interior of each R_i. Consider the class of functions $\Phi = (\phi_1, \phi_2, \ldots, \phi_n)$ where each ϕ_j vanishes on the boundary of R and is continuous along with its d_j-weighted normal derivatives on the boundaries of the R_i and is C^2 in the interior of the subregions R_i, $1 \leqq i \leqq m$. Define $T\Phi = \Psi$ where $\psi_j = \operatorname{div} d_j \operatorname{grad} \phi_j + \sum_{k=1}^{n} a_{jk} \phi_k$, $1 \leqq j \leqq n$. Then with certain arithmetic restrictions on the parameters (which assure the transitivity of T) the kinetics equations become (1) $\partial\Phi/\partial t = T\Phi$. The authors show that the spectrum of T is discrete and that T possesses a unique non-negative eigenfunction corresponding to a simple, real eigenvalue, algebraically larger than the real part of all other eigenvalues of T. This result is obtained from the Krein-Rutman theorem, applied to $I - \exp[t(T - \rho I)^{-1}]$ for large positive ρ. Other spectral theorems are shown for T and these results are applied to the study of equation (1). (Received March 3, 1958.)

544-29. Anil Nerode: *Remarks on isolic arithmetic*.

For notation, see J. C. E. Dekker, Abstracts 63-5-610, 538-67. For n a positive integer, let $\lambda(n)$ be the largest i such that for some prime p, p^i divides n. Let $\phi(n)$ be the euler totient. Then for all isols X, $X^{\phi(n)+\lambda(n)} \equiv X^{\lambda(n)}$ (mod n). If $F(X_1,\ldots,X_k)$ is a polynomial with integral coefficients, p a finite prime, and $F(X_1,\ldots,X_k) \equiv 0$ (mod p) for all rational integers $X_1,\ldots,X_k$, then $F(X_1,\ldots,X_k) \equiv 0$ (mod p) for all isolic integers $X_1,\ldots,X_k$. The Chinese remainder theorem holds for isolic integers when the moduli are finite. For all isols X, Y, $(X!)^Y$ divides $(XY)!$; this and Dekker's result that $X!Y!$ divides $(X+Y)!$ follow from the lemma below. If p is a permutation of the positive integers ϵ, $\pi(p) = [x \mid px \neq x]$. The family of p with $\pi(p)$ finite can be generated without repetitions in a sequence $p_0, p_1, \ldots$, where $p_n(x)$ is recursive in n,x. For $\sigma \subset \epsilon$, put $\sigma! = [n \mid \pi(p_n) \subset \sigma]$. If E is an equivalence relation on ϵ, let $P(E) = [n \mid xEp_n(x)$ for all x in $\epsilon]$. *Lemma*. If E has a recursive characteristic function and σ is isolated, the isol represented by $\sigma! \cap P(E)$ divides the isol represented by $\sigma!$. (Received March 3, 1958.)

544-30. Dieter Puppe: *A theorem on semi-simplicial monoid complexes*.

THEOREM. Let $M = (M_q)$, $q = 0,1,\ldots$, be a (complete semi-simplicial) monoid complex, $G = (G_q)$ a group complex and $f\colon M \to G$ a semi-simplicial homomorphism. Assume (1) $G_q = f(M_q)\cdot f(M_q)^{-1}$ for all q, (2) if $f(a) = f(b)$, $a,b \in M_q$ then $ax = bx$ for some $x \in M_q$, (3) $|M|$ is connected. Then the geometric realization $|f|\colon |M| \to |G|$ is a homotopy equivalence. For every *abelian* monoid complex M there exists one and up to isomorphisms only one group complex G (which is automatically abelian) together with a semi-simplicial homomorphism $f\colon M \to G$ having properties (1) and (2). In this case the theorem has also been proved by E. H. Spanier. As an application one can give a new proof for $\pi_q(SP(X)) \cong H_q(X)$ ($q > 0$, X = connected CW-complex, SP(X) = infinite symmetric product), in which the isomorphism is described more explicitly than in the original proof of A. Dold and R. Thom (*Quasifaserungen und unendliche symmetrische Produkte*, Ann. of Math. vol. 67 (1958). (Received March 3, 1958.)

544-31. E. C. Posner: *Algebraic sets and finitely generated semigroups*.

Let K be a field and n a positive integer, both fixed throughout. Let G be a finitely generated semigroup, and let $s_1,\ldots,s_r$ be r generators of G. Let V_r be the product of r copies of M_n, the n × n matrices over K. Let $W(G; s_1,\ldots,s_r)$ be the set of r-tuples $(w_1,\ldots,w_r) \in V_r$ such that $w_i = \phi(s_i)$, $1 \leqq i \leqq r$, for some semigroup homomorphism $\phi: G \rightarrow M_n$. W is an algebraic set, since a mapping ϕ from the s_i into a set $\{w_i\}$, $1 \leqq i \leqq r$, can be extended to a homomorphism of all of G if and only if the w_i satisfy the same relations among themselves that the s_i do. Furthermore, if $t_1,\ldots,t_q$ is another finite set of generators of G, then in the obvious sense (a W need not be irreducible) and in the obvious way, $W(G; s_1,\ldots,s_r)$ and $W(G; t_1,\ldots,t_q)$ are birationally biregularly equivalent over K. This construction can be used to give an immediate proof of the following generalization of a theorem of Malcev: A finitely generated semigroup of matrices is not isomorphic to a proper homomorphic image of itself. (Received March 3, 1958.)

544-32. L. F. McAuley: Concerning an unnecessary condition in certain upper semicontinuous decompositions of E^3 into itself.

Certain u.s.c. decompositions G of E^3 into E^3 have been given that require the collection H of nondegenerate elements of G be such that H^* is a G_δ set. This condition may be removed from several recent theorems due to the author. One now reads as follows. Suppose that G is an u.s.c. collection of straight line intervals and points filling up E^3 and that $\{P_i\}$ is a sequence of planes such that an interval in G is perpendicular to and intersects P_n for some n. Then the decomposition space is topologically E^3. An application of this theorem answers in the affirmative the following question raised by R. H. Bing and Deane Montgomery. Does each u.s.c. collection G of straight line intervals and points filling up E^3 such that each interval in G is parallel to one of the coordinate axes yield a hyperspace homeomorphic to E^3? (Received March 3, 1958.)

544-33. M. W. Hirsch: Criteria for immersibility of manifolds.

Let M and A be C^∞ differentiable manifolds of dimensions k and n respectively. If $f: M \rightarrow A$ is a regular map, $f_*: T_k(M) \rightarrow T_k(A)$ denotes the induced map, where T_k denotes the bundle of k-frames. f_* is equivariant with respect to the action of GL(k,R). THEOREM 1: If $k < n$, $f \rightarrow f_*$ induces a one-one correspondence between the set of regular-homotopy classes of regular C^∞ maps $f: M \rightarrow A$ and the set of equivariant-homotopy classes of equivariant maps $T_k(M) \rightarrow T_k(A)$. THEOREM 2: Let E^m denote Euclidean m-space. If $k < n$ and M is immersible in E^{n+r} with a field of normal r-frames, then M is immersible in E^n. Examples: Let P^k be projective k-space. P^3 is immersible in E^4, P^6 in E^7, P^7 in E^8, P^9 in E^{15}. If $k = 2^i - 2$ and P^k is immersible in E^{k+2}, then P^{k+1} is parallelizable. If M^k is immersible in some E^n with a trivial normal bundle, then M^k is immersible in E^{k+1}; in particular, a parallelizable M^k is immersible in E^{k+1}. Every M^5 is immersible in E^8; in general, if $k \equiv 1 \bmod 4$, M^k is immersible in E^{2k-2} if and only if $\overline{W}^{k-1}(M^k) = 0$. M^4 is immersible in E^6 if and only if there is an element b in $H^2(M^4)$ such that $b \equiv \overline{W}^2(M^4) \bmod 2$ and $b^2 = \overline{P}^4(M^4)$. The methods are those described in Amer. Math. Soc. Notices, February, 1958, Abstract 539-19. A theorem of Massey (Amer. Math. Soc. Notices, February, 1958, Abstract 538-17) is also used. (Received March 4, 1958.)

544-34. W. C. Taylor: Numerical solution of algebraic equations.

In order to solve the system of simultaneous equations (1) $f_i(x) = 0$, $i = 1,2,\ldots,n$, for x: $x_1, x_2, \ldots, x_n$, it would suffice to obtain a representation (for brevity, introducing $x_0 = 1$ and employing the summation convention) (2) $f_i(x) \equiv P_{ij}(x)x_j$ with the property that the solution x of (3) $P_{ij}(y)x_j = 0$ is independent of y. The following working principle is useful but admittedly not precise: If the solution x of (3) is only approximately independent of y, then (3) affords an iterative procedure in which an estimated solution of (1) is inserted as y in (3) and the improved estimate x is obtained at the cost of solving the system of linear equations. The present communication is concerned chiefly with algebraic equations in the case $n = 1$. For computing a real root of an algebraic equation there is described a numerical method which consists of an approximate factorization as in (2) by a relaxation method and subsequent iteration. Analogous methods are available for quadratic or higher degree factors. For $n > 1$, the corresponding methods are less determinate and perhaps less interesting. (Received March 4, 1958.)

544-35. G. H. Meisters: On almost periodic solutions of a class of differential equations.

Let S denote complex, finite dimensional vector space. Let $\phi(t)$ be a function with values in S defined and differentiable for all t on the real line R. Let $F(x,t)$ be a function on $Q \times R$ into S where Q is the range of $\phi(t)$. Theorem. If (1) $\phi(n)$, $n = 0, \pm 1, \pm 2, \ldots$, is an almost periodic sequence, (2) $d\phi(t)/dt = F(\phi(t),t)$ for all t in R, (3) $F(x,t)$ is Bohr almost periodic in t for each x in Q and (4) $\|F(x,t) - F(y,t)\| \leqq K\|x - y\|$ for all t in R and all x,y in Q (where $\|x\|$ denotes the norm of x), then $\phi(t)$ is a Bohr almost periodic function. (Received March 4, 1958.)

544-36. C. E. Langenhop: *Mean functions of almost periodic functions on a group*. Preliminary report.

Let f(x) be an almost periodic function in the sense of von Neumann (Trans. Amer. Math. Soc. vol. 36 (1934) pp. 445-492) with x in a group G. Then $\phi(x,a) = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} f(xa^i)$, the convergence being uniform in G, is almost periodic as a function of x and satisfies $\phi(xa,a) = \phi(x,a)$ for all x in G. If a is in the center of G and A denotes the subgroup consisting of all powers of a, then this periodicity enables one to define a function $\Phi(\xi)$ on the group G/A by the equation $\Phi(\xi) = \phi(x,a)$ where x is any element of the coset corresponding to ξ. The function Φ is almost periodic on G/A and $M_\xi[\Phi] = M_x[\phi] = M_x[f]$ where M denotes the mean. (Received March 4, 1958.)

544-37. Newcomb Greenleaf and R. J. Wisner: *On unique factorization in semigroups of the integers*.

Let a,b be fixed integers with $a > 0$, $b \geqq 0$; and let $S = \{ax + b \mid x \in N\}$, $T = \{ax + b \mid x \in I\}$, where N denotes the natural numbers and I denotes the integers. Demand that S and T be closed with respect to multiplication, so that they are semigroups. The authors were led to a study of unique factorization in such semigroups, called affine semigroups, by an example (given by H. Rademacher): the numbers 1,4,7,10,13,... of the form $3x + 1$ possess 4,10,25 as primes, and $100 = 10 \cdot 10 = 4 \cdot 25$. Elementary theorems from the theory of numbers and the theorem of Dirichlet that the progression $\{ax + b\}$ contains infinitely many primes if $(a,b) = 1$ are used to prove: (1) S has unique factorization if and only if $b = 1$ and $a = 1$ or 2, and (2) T has unique factorization if and only if $b = 1$ and $a = 1,2,3,4$, or 6. (Received March 4, 1958.)

544-38. Berthold Schweizer and Abe Sklar: *Statistical metric spaces arising from sets of random variables*.

Statistical metric spaces are generalizations of ordinary metric spaces in which, instead of a number, a distribution function F_{pq} is associated with each pair of points p,q. The number $F_{pq}(x)$ may be interpreted as the probability of the distance between p and q being $\leqq x$. Such spaces arise, for example, from sets of random variables (p,q,...) on an ordinary metric space M if F_{pq} is taken to be the distribution function of the random variable d(p,q), where d is the metric in M. A case of particular interest results when p,q,... are independent spherical Gaussian random variables on Euclidean n-space. In the resulting statistical metric spaces the functions F_{pq} are the well-known 'spherical' or 'generalized Rayleigh' distribution functions. Among the geometric consequences of the definition of these spaces are the following: the means of the functions F_{pq} define a metric which is asymptotic to the 'underlying' n-dimensional Euclidean metric at large distances, but drastically nonEuclidean in the small; the functions F_{pq} themselves satisfy various special cases of the generalized triangle inequality of Menger (Proc. Nat. Acad. Sci. vol. 28 (1942) pp. 535-537). (Received March 4, 1958.)

544-39. Donald Austin: *On the interdependence of the Kolmogorov differential equation*.

Let $\{p_{ij}(t)\}$ be a stationary Markov transition matrix. Assume that the diagonal elements possess finite derivatives at 0 and that the matrix possesses no identically 0 functions. It is shown that if the Kolmogorov differential equation $p'_{ij}(t) = \sum_k p'_{ik}(0)p_{kj}(t)$ holds for some i,j and t then it holds for that i, for all j and t. (Received March 5, 1958.)

544-40. S. T. Hu: *On continuous associative multiplications in arcwise connected compact Hausdorff spaces*.

In a given topological space X, a continuous associative multiplication with two-sided unit which fails to make X a group will be simply called an essential multiplication. Assume that X is an arcwise connected compact Hausdorff space in which an essential multiplication is given with a point u as its two-sided unit. It is well-known that the set H of all points of X with two-sided inverse forms a compact topological group and that the complement $J = X \setminus H$ is the maximal proper two-sided ideal of X. As a preliminary result in the present paper, it is proved that H is arcwise accessible from J. For each point x of X, the tangent space T(X,x) is defined to be the subspace of the space X^I of all paths in X which consists of the paths $\sigma: I \rightarrow X$ such that $\sigma(t) = x$ if and only if $t = 0$. If X is triangulable, then T(X,x) is of the same homotopy type as the boundary of the star of x in X. The main result in this paper is that T(X,x) is contractible to a point if x is in the subspace H of X. This implies Wallace's theorem that every closed manifold admits no essential multiplication as an immediate consequence. (Received March 5, 1958.)

544-41. W. B. Jurkat: *On semigroups of positive matrices*. II.

This paper continues the analytic investigations of "generalized Markov processes", i.e. matrix-functions $P(t) = (p_{jk}(t))$ satisfying $P(t) \geqq 0$, $P(s + t) = P(s)P(t)$ for $s,t > 0$ and $P(t) \to I$ as $t \to +0$. Suppose that $p'_{jk}(0) = q_{jk}$ is finite for all j,k (or if $j = k$ only). The condition (i) $\sum_m p_{jm}(s)|q_{mm}|p_{mk}(t) < +\infty$ for all j,k, s, t implies the Kolmogorov differential equations $P'(t) = QP(t) = P(t)Q$ for $t \geqq 0$. Conditions of the same type also imply a certain degree of uniqueness for solutions of the Kolmogorov equations. In particular it follows from (i) that P(t) must be Feller's minimal process associated with Q. In the case of ordinary Markov processes where all row-sums are less or equal to 1, the slightly stronger condition (ii) $\sum_m p_{jm}(s)|q_{mm}| < +\infty$ for all j,s implies that there is no other ordinary Markov process satisfying $P'(t) = QP(t)$ for $t \geqq 0$ but the minimal process. This now explains why also $P'(t) = P(t)Q$ must hold for $t \geqq 0$. (Received March 5, 1958.)

544-42. Mark Mahowald: *A summability theorem in countable toral groups*.

Let G be a countable toral group. Then the representation of a function as an expansion with respect to the characters is Abel summable at points of continuity of the function. (Received March 5, 1958.)

544-43. WITHDRAWN.

544-44. Dana Scott: *Definability in polynomial rings*.

Let K be a field of characteristic 0 and let K[x] be the polynomial ring over K in one variable. R. M. Robinson has shown (Trans. Amer. Math. Soc. vol. 70 (1951) pp. 137-159) that the predicate of being a rational integer in K is definable in the first-order theory of the ring K[x]. The author has shown by a similar method that the predicate of being an algebraic number over the prime field of rationals is also definable in the theory of K[x]. Hence, if K is the field of complex algebraic numbers and C is the field of all complex numbers, then, even though K and C have the same first-order theories, the rings K[x] and C[x] do not. This answers a question of A. Robinson. If L is any algebraically closed field of characteristic 0, it is shown further that the theory of L[x] depends on the transcendence degree of L over the prime field. Exactly similar results have been obtained by considering directly the second-order monadic theories of the fields themselves, rather than involving the polynomial rings in the definitions. (Received March 5, 1958.)

THE APRIL MEETING IN STANFORD, CALIFORNIA

April 18-19, 1958

545-1. V. W. Bolie: Divergence from Bessel's integral in a radiation equation.

This paper concerns the mathematical exploration of an equation representing the composite radiation from a circular array of isotropic sources whose excitations are phased sinusoidally with their peripheral disposition. The equation is in the form of a series of terms, each of which corresponds to a particular source on the circle. In the limit, as the number of sources increases without bound, the series reduces to Bessel's Integral. Of particular interest is the case in which the number of sources is large but finite. The functional behavior of the equation is studied as the number of sources is varied. The divergence from the Bessel integral is graphically portrayed from numerical calculations. (Received February 13, 1958.)

545-2. F. P. Peterson: Some remarks on complex n-plane bundles.

It is well known that the set of topological equivalence classes of complex n-plane bundles over a space M is in 1-1 correspondence with the set of homotopy classes of maps from M into the complex Grassmannian manifold, $G_n(C)$. Using the results of Bott (Proc. Nat. Acad. Sci. vol. 43, p. 933), we study this set of homotopy classes by constructing the Postnikov system of $G_n(C)$. For example, we can prove the following results. (1) If dim $M \leqq 2n$, then the set of equivalence classes of complex n-plane bundles is an abelian group. (2) If dim $M \leqq 2n$ and assuming that $H^*(M)$ has no torsion in even dimensions, then a bundle is trivial if and only if all its Chern classes vanish. (3) In low dimensions, we can make explicit computations of the above group (or set when dim $M > 2n$). (4) We can use the same techniques to study the problem of what elements in $H^*(M)$ can be Chern classes of complex n-plane bundles. (Received February 14, 1958.)

545-3. J. B. Rosen: Fourier series approximation to nonlinear parabolic boundary value problems.

The nonlinear parabolic partial differential equation $L[u] = u_t$ -

$F[u_{xx}, u_x, u, x, t] = 0$ (*), is considered in the region $0 < x < 1$, $0 \leqq t \leqq T$, with boundary conditions $b_\ell u(\ell, t) + b'_\ell u_x(\ell, t) = c_\ell(t)$, $\ell = 0, 1$, and the initial condition $u(x, 0) = w(x)$. b_ℓ and b'_ℓ are constants, $c_\ell(t) \in C^1$ and $w(x) \in C^2$. An approximate solution $U(x,t) = \sum_{i=0}^{N-1} a_i(t) g_i(x)$ is considered where the $g_i(x)$ are the first N functions of an orthonormal, complete set chosen in advance. The requirement that $\sum_{j=1}^{M} L^2 [U(jh, t)] = \min.$, where $1/h = M + 1 > N$, and that $U(x,t)$ satisfies the boundary conditions reduces (*) to a system S_N of N ordinary differential equations determining the $a_i(t)$. It is proved for the linear case that L_2 convergence of $U(x,t)$ to $u(x,t)$ is obtained as $N \to \infty$, with $M = \sigma N^2$, $\sigma = \text{const.} > 0$ For finite $M > N$, a matrix method suitable for a high speed computer has been developed in detail for obtaining the system S_N from (*). For F linear with variable coefficients depending only on x, S_N is a linear constant coefficient system. An approximate analytic solution to (*) is then obtained by a single eigenvalue and eigenvector calculation. For a nonlinear F the nonlinear system S_N is obtained and efficient methods for its numerical solution described. An error estimate is obtained as part of the method. (Received February 17, 1958.)

545-4. R. F. Dennemeyer: Elliptic forms in Hilbert space, and second order elliptic differential equations.

Let T be a region of class K of Calkin and Morrey (Duke Math. J. vol. 6 1940), $t = (t_1, \ldots, t_n) \in T$. After defining a suitable Hilbert space of functions $x(t)$, with subspace $\mathcal{Q} = \overline{C}_0^\infty$, we consider the quadratic form $Q(x) = D(x) + K(x)$, where $D(x)$ is positive definite and $K(x)$ is w-continuous. Associated with $Q(x)$ is the Euler differential operator $E(x) = (\partial/\partial t_i)(R_{ij}(\partial x/\partial t_j)) - x(P - \partial Q_i/\partial t_i)$. Basing the development on the very general theory of quadratic forms and indices given by M. R. Hestenes (Pacific J. Math. vol. 1, no. 4, 1951), we attempt an extension to multiple integrals of the highly developed theory of conjugate points for simple integral problems of the calculus of variations. Existence, compatibility, and uniqueness theorems are proven (without use of Green's functions) for the problem $E(x) = f(t)$, $x(t) = \phi(\sigma)$ on T^*, the boundary of T, for various cases of the vanishing and non-vanishing of f and σ. Let $\mathcal{Q}(\lambda)$, $\lambda' \leqq \lambda \leqq \lambda''$, be an expanding family of subspaces of $\mathcal{Q}$, with $T(\lambda)$ a corresponding family of expanding subsets of T. Theorems are obtained on the index $i(\lambda)$, nullity $n(\lambda)$, and on the number of conjugate hyper-surfaces within T. Examples are given to illustrate the theory. (Received February 21, 1958.)

545-5. R. M. Baer and Paul Brock: Natural sorting.

The problem considered is that of finding the expected length of a maximal sorted subsequence within a sequence of random positive integers. The function $P(n,M,R)$ is defined as the totality of sequences of length M with maximal element R, containing a monotonic nondecreasing subsequence of length n or larger. Although the general distribution is not obtained, results are obtained for $P(n,M,2)$ and for the case where repetitions of elements in the original sequences are not permitted. (Received February 24, 1958.)

545-6. Seymour Goldberg: Linear operators and their conjugates.

Let X and Y be normed linear spaces and T a linear operator with domain D dense in X and range $R \subset Y$. T' is to denote the conjugate of T. For $D = X$ and T continuous, Taylor and Halberg (J. Reine Angew. Math. vol. 198 (1957) pp. 93-111) systematically presented theorems about the range and inverse of one of the operators T,T' which result from an hypothesis about the other. It is shown that the same results still hold if the assumptions on T are weakened to: T closed and $\overline{D} = X$. Sample theorem (T closed, $\overline{D} = X$): Let X be complete and let T' have a bounded inverse. Then $R = Y$. Moreover, if T^{-1} exists, it is continuous. It is further shown that even when the restriction that T be closed is removed, most of the theorems are still valid. Examples are given to show which theorems no longer hold. (Received February 24, 1958.)

545-7. A. E. Taylor: The norm of a real linear transformation in Minkowski space.

Let $\ell^p(n)$ be the Minkowski space of vectors $x = (\xi_1,\dots,\xi_n)$ with the usual norm $\|x\|_p$ defined as a pth root of a sum of pth powers, where $p \geqq 1$. Let A be a linear transformation of $\ell^p(n)$ into $\ell^q(m)$ and suppose the matrix representing A (where we use the standard bases in the two spaces) has real elements. If we confine attention to vectors with real components, A has a certain norm $\|A\|_r$. When vectors with complex components are allowed, we denote the norm of A by $\|A\|_c$. It is then a theorem that $\|A\|_r = \|A\|_c$ if $q \geqq p$. This proposition was announced by M. Riesz in his famous 1927 paper on convexity and bilinear forms. The proof was barely sketched, and the intended argument has seemed obscure to some people. It is the purpose of this paper to record a proof in full. The following lemma plays the key role: Suppose x and y are linearly independent vectors in the real space $\ell^p(n)$, and u,v are two vectors, not both zero, in the real space $\ell^q(m)$. Let $F(s,t)$ be the quotient of $\|u + (s+it)v\|_q$ by $\|x + (s+it)y\|_p$, where s and t are real variables. Then, if $q > p$, F cannot have a relative maximum at the point $s = 0$, $t = 1$. (Received February 24, 1958.)

545-8. David Gale: Transient flows in networks.

A dynamic network N is a graph in which corresponding to each edge is a non-negative real valued function giving the capacity of the edge as a function of time. N is assumed to have two distinguished nodes s and s', called *source* and *sink*. Let μ_k denote the maximum amount which can be shipped from s to s' in k time periods by means of some flow ϕ_k which is zero at time $t = 0$. It is shown that there exists a *universal maximal flow*, that is, there exists a single flow $\bar{\phi}$ which is zero at $t = 0$, and such that the amount which flows from s to s' in k time periods is μ_k for all k. (Received February 26, 1958.)

545-9. W. W. Comfort: Šilov boundaries induced by certain Banach algebras.

Let G be a commutative semigroup and $\ell_1(G)$ the Banach algebra described by Hewitt and Zuckerman (Trans. Amer. Math. Soc. vol. 83 (1956) pp. 70-97). Let $\hat{G}$ be the set of all semicharacters of G, and suppose that $\hat{G}$ separates points of G, so that $\ell_1(G)$ is semisimple. For each element α of $\ell_1(G)$, define $\hat{\alpha}$ on $\hat{G}$ by $\hat{\alpha}(\chi) = \sum_{x \in G} \alpha(x)\chi(x)$. Let ∂ denote the Šilov boundary induced in $\hat{G}$ (with the Gel'fand topology) by the algebra $[\ell_1(G)]^\wedge$. Let $B = \{\chi \in \hat{G} \mid |\chi| = 1\}$, $\Gamma = \{\chi \in \hat{G} \mid |\chi| = 0 \text{ or } 1\}$. Among the results obtained are the following: $B \subset \partial \subset \Gamma$. If G is a union of groups (in particular, if every element of G is idempotent), then $\Gamma = \hat{G} = \partial$. For χ_1, χ_2 in Γ, write $\chi_1 \sim \chi_2$ if there exists $\chi \in B$ with $\chi\chi_1 = \chi_2$. Then ∂ is the union of those $\sim$-equivalence classes which it meets. If $\chi \in \Gamma$, write $G_i = \{x \in G \mid |\chi(x)| = i\}$, $i = 0,1$. Let G'' be the semilattice defined by Hewitt and Zuckerman (loc. cit. p. 78). If χ is isolated in Γ, then there exists a finite group $x'' \in G''$ for which $x''G = x''$, and $\chi \in \partial$ if and only if the only extension in Γ of $\chi|G_1$ to each nongroup $y'' \in G''$ covered by x'' is 0. (Received February 27, 1958.)

545-10. R. J. Blattner: Automorphic group representations.

Let $\mathfrak{A}$ be the ring of the Clifford distribution of variance 1 over the real Hilbert space $\mathcal{H}$, and let Γ be the canonical representation of the orthogonal group on $\mathcal{H}$ as $*$-automorphisms of $\mathfrak{A}$. Let G^+[resp. G^-] be the set of orthogonal transformations U on $\mathcal{H}$ such that I - U [resp. I + U] is Hilbert-Schmidt and the eigenspace of U [resp. -U] belonging to -1 is of even [resp. odd] dimension. Set $G_0 = G^+ \cup G^-$. THEOREM 1: *Γ(U) is inner if and only if* $U \in G_0$. A representation ρ of a topological group G into the *-automorphic group A of a ring of operators $\mathfrak{A}$ is called *continuous* if the maps $g \rightarrow T^{\rho(g)}$, $T \in \mathfrak{A}$, are continuous from G into $\mathfrak{A}$ in the weak topology. THEOREM 2: *Let ρ be a nontrivial continuous representation of an open simple Lie group* G *as automorphisms of a finite ring $\mathfrak{A}$. Then ρ(g) is inner only if* g *is central*. (Received February 28, 1958.)

545-11. C. W. Clark: *Asymptotic distribution of the eigenvalues for the lower part of the one-dimensional Schrödinger operator spectrum*.

Consider the eigenvalue problem associated with $u''(x) + [\lambda - q(x)]u(x) = 0$, $0 < x < +\infty$, $u \in L_2(0,\infty)$, $u(0^+) = 0$. For $N(\lambda) = \sum_{\lambda_j \leqq \lambda} 1$, the number of eigenvalues $\leqq \lambda$, the formula $N(\lambda) = \pi^{-1}\int_{q(x) \leqq \lambda} [\lambda - q(x)]^{1/2}dx + O(1)$ is well-known under various conditions on q(x), all of which assume $q(x) \rightarrow +\infty$ as $x \rightarrow +\infty$, thereby assuring a pure point spectrum. It is shown that the formula remains valid for the negative eigenvalues $\{\lambda_j\}_{j=1}^{\infty}$ in certain cases where q(x) has a finite limit as $x \rightarrow +\infty$. Specifically, the formula can be proved provided that $q(x) \uparrow 0$ and is 5 times continuously differentiable on some interval $[x_0, +\infty)$, and satisfies, as $x \rightarrow +\infty$ (i) $|q(x)|^{-1} = O(x^{2-\eta})$ for some $\eta > 0$; (ii) $|q^{(n)}(x)|/|q^{(n-1)}(x)| = O(x^{-1})$, $n = 1,2,3,4,5$; (iii) $|q^{(n-1)}(x)|/|q^{(n)}(x)| = O(x)$, $n = 1,2,3$. In addition, q(x) must satisfy such conditions on $0 < x < x_0$ that the maximum-minimum principle holds for the eigenvalues, and that the eigenvalues for the corresponding problem on $(0,x_0)$ eventually become positive (a sufficient condition [Friedrichs, Math. Ann. vol. 112 (1935) pp. 1-23] being $q(x) \geqq a - bx^{-2}$ for some $b < 1/4$). (Received February 28, 1958.)

545-12. E. E. Kohlbecker: *Asymptotic properties of the sum of the divisors of an integer.*

Let $a_1 < a_2 < \ldots$ be a sequence of positive integers (denoted by A) such that $n(u) \sim u^{\omega}L(u)$ as $u \rightarrow \infty$, where n(u) denotes the number of elements of A which are less than or equal to u, ω is a positive constant and L(u) is a slowly oscillating function in the sense of Karamata. Let $\sigma(k,A)$ denote the sum of the divisors of k which are terms of A, and let S(u) be the summatory function of $\sigma(k,A)$. It is shown that $S(u) \sim (\omega/(\omega + 1))\zeta(\omega + 1)u^{\omega+1}L(u)$ as $u \rightarrow \infty$, where $\zeta(\omega + 1)$ is the Riemann Zeta-function. (Received March 3, 1958.)

545-13. R. C. Gilbert: *Generalized resolvents of a symmetric operator*.

Let T be a closed symmetric operator in a Hilbert space H. If $\tilde{T}$ is a self-adjoint extension of T, in a possibly larger Hilbert space $\tilde{H}$, let $\tilde{R}(\lambda)$ be its resolvent. If $\tilde{P}$ is the projection of $\tilde{H}$ onto H, $R(\lambda) = \tilde{P}\tilde{R}(\lambda)$ is called a generalized resolvent of T. The set $\mathcal{R}$ of all generalized resolvents of T is a convex set. If Im $\lambda_0 > 0$, the elements of $\mathcal{R}$ are in a 1 - 1 correspondence with the set $\mathcal{F}$ of all operators $F(\lambda)$ mapping $H \ominus$ range $(T - \lambda_0 E)$ into $H \ominus$ range $(T - \bar{\lambda}_0 E)$ which are analytic for Im $\lambda > 0$ and satisfy $\|F(\lambda)\| \leqq 1$. It is shown that, if T has finite and equal defect indices, the set $\mathcal{R}_0$ of all $R(\lambda)$ for which $\dim(\tilde{H} \ominus H) < \infty$ is dense in $\mathcal{R}$, in the sense that for each $R(\lambda)$ in $\mathcal{R}$ there is a sequence $\{R_n(\lambda)\}$ from $\mathcal{R}_0$ which converges in norm to $R(\lambda)$ for each λ. The proof is obtained by showing that the set $\mathcal{F}_0 \subset \mathcal{F}$ corresponding to $\mathcal{R}_0$ is dense in $\mathcal{F}$ in the same sense. It is known that the elements of $\mathcal{R}_0$ are extreme points of $\mathcal{R}$. Application is made to symmetric ordinary differential operators. (Received March 3, 1958.)

545-14. L. K. Barrett: *On n-regular curves*.

A necessary and sufficient condition that a continuum (a compact connected set in a separable metric space) be a {regular curve} {dendrite} {n-regular curve} is that any two of its points can be separated by {a finite set of points} {one point} {a set of n points}. In this note a complete classification is made of n-regular curves. As a corollary to Theorem 1.1 of (L. K. Barrett, *Regular curves and regular points of finite order*, Duke Math J. vol. 22 (1955) pp. 295-304) if M is an n-regular curve some set of less than n points must separate M. It is here shown that the set of points of M of order $\leqq$ n have the properties of such points of a continuum of bounded order n. The set of local separating points of order greater than n is countable (G. T. Whyburn, *Local separating points of continua*, Monatsh. fur Math. u. Phys. vol. 36 (1929) pp. 305-314). Based on a set of lemmas concerning properties of a sequence of cuttings, it is shown that the set N of non local separating points of order greater than n is countable and that N contains no limit point of itself. (Received March 3, 1958.)

545-15. Leonard Gillman and Meyer Jerison: *Homomorphisms of rings of continuous functions*.

Any homomorphism u of the ring C(Y) of all continuous real-valued functions on a completely regular space Y into a ring C(X) may be represented as follows: Let e denote the constant function 1 on Y, and let E be the subset of X where ue(x) = 1. Then there exists a unique continuous mapping t from E into the Hewitt space νY [Trans. Amer. Math. Soc. vol. 64 (1948) pp. 45-99] such that for all g in C(Y), (ug)(x) = $g^{\nu}(tx)$ if x belongs to E and (ug)(x) = 0 otherwise. Here, g^{ν} is the unique continuous extension of g to Y. As a consequence, the ring homomorphism u is also a lattice homomorphism. (Received March 3, 1958.)

545-16. C. E. Burgess: *Homogeneous plane continua which are circularly chainable and contain an arc*.

A compact connected metric space is said to be circularly chainable if for each positive number ϵ there exists a finite sequence of open sets $D_1, D_2, \ldots, D_n$ covering M such that (1) each D_i has a diameter less than ϵ and (2) $D_i \cdot D_j \neq 0$ if and only if $|i - j|$ is either 0,1, or n - 1. Let K denote a homogeneous bounded plane continuum which is circularly chainable and does not have more than two complementary domains. Bing and Jones have described such a continuum which separates the plane and is different from a simple closed curve [Bull. Amer. Math. Soc. vol. 60 (1954) Abstracts 766 and 770]. The main purpose of this paper is to show that K is a simple closed curve provided it contains an arc. (Received March 3, 1958.)

545-17. L. N. Howard: *On hydrodynamic stability of a wake*.

The Orr-Sommerfeld problem for stability of an approximation to a wake in unlimited viscous two-dimensional flow (steady velocity profile: $w = A - e^{-y^2}$) is attacked by the integral equation method used previously for the jet problem (Notices Amer. Math. Soc. vol. 5, no. 1, Abstract 539-24). The proof that the neutral curve is bounded away from R = 0 in the α - R plane goes through as in the jet case, and in fact holds more generally for any symmetric velocity profile which approaches zero (or a constant) properly at infinity. For large Reynolds' number, the same behavior as in the jet problem, $\alpha \sim kR^{-2}$, is found for the neutral curve. (Received March 3, 1958.)

545-18. Morgan Ward: *Arithmetical properties of Bernoulli polynomials*.

The arithmetical properties of Bernoulli numbers due to von Staudt, Adams, Voronoi and Kummer are extended to Bernoulli polynomials with a fixed rational argument. Various arithmetical properties of the Euler, Genocchi numbers due to Lucas and others appear as special cases. (Received March 3, 1958.)

545-19. H. P. Kramer: *Symmetrizable Markov matrices*.

Suppose that the evolution of the state probabilities $p(t) = \{p_1(t), \ldots, p_n(t)\}$ of a discrete state, continuous parameter Markov process is governed by the system of differential equations $dp/dt = Qp$ where Q is the $n \times n$ matrix of transition rates with elements (ij). For a large class of processes, including birth and death processes, an explicit formula for $\pi_m = \lim p_m(t)$ as $t \to \infty$ is given in *Theorem* 1: Suppose no states are isolated and for every closed loop of states $i, k_1, k_2, \ldots, k_s$, $i(ik_1)(k_1k_2)\ldots(k_si) = (ik_s)(k_sk_{s-1})\ldots(k_1i)$. Then $\pi_f = (fk_1)(k_1k_2)\ldots(k_si)\pi_i/(ik_s)\ldots(k_1f)$ where $i, k_s, \ldots, k_1, f$ represents a chain of states with nonvanishing transition rates between adjacent members of the chain. *Theorem* 2: If the conditions of Theorem 1 are satisfied, then the characteristic values of Q, which are the decay coefficients of the process, are real and nonpositive. By variational methods estimates can be obtained for the largest nonzero characteristic values and thus also for the rate of approach of the state probabilities to their equilibrium values. (Received March 4, 1958.)

545-20. Berthold Schweizer and Abe Sklar: *On triangle inequalities for statistical metric spaces*.

Statistical metric spaces, defined by K. Menger (Proc. Nat. Acad. Sci. vol. 28 (1942) pp. 535-537) are generalizations of ordinary metric spaces in which, instead of a number, a distribution function F_{pq} is associated with each pair of points p,q. The value $F_{pq}(x)$ may be interpreted as the probability of the distance between p and q being $\leqq x$. The functions F_{pq} are assumed to satisfy: (1) $F_{pp}(x) = 1$ for $x \geqq 0$ (2) $F_{pq}(x) = 0$ for $x < 0$ (3) $F_{pq} = F_{qp}$ (4) $T(F_{pq}(x), F_{qr}(y)) \leqq F_{pr}(x + y)$ for any triple of points p,q,r and any $x,y > 0$; T is a 2-place function defined for $0 \leqq s, t \leqq 1$, satisfying: (a) $0 \leqq T \leqq 1$ (b) $T(u,v) \geqq T(s,t)$ for $u \geqq s$, $v \geqq t$ (c) $T(s,t) = T(t,s)$. This function, T, which appears in the generalized triangle inequality (4), is not further specified by Menger. Recent investigations have shown that natural choices for T, in order of increasing stringency, are the 2-place functions Product, Min, Max, and Sum-Product. By postulating that $T(T(a,b),c) = T(a,T(b,c))$, (4) may be extended to a polygonal inequality. In a wide variety of cases, (4) can be shown to hold with some one of the listed choices of T. For example, if $F_{pq}(x) = H(x/d(p,q))$, where d is a 2-place function satisfying the conditions for an ordinary metric, then (4) holds with $T = \text{Min}$. (Received March 4, 1958.)

545-21. V. L. Klee, Jr.: Remarks on the geometry of normed linear spaces.

(I) For a bounded subset X of a metric space Y, the radius of X in Y (Denoted by $r_Y X$) is the greatest lower bound of numbers r such that for some $p \in Y$, $X \subset S(p,r)$. When X is a convex set in a normed linear space E, two cases are of special interest and importance — $Y = X$ and $Y = E$. It is proved that the following two assertions are equivalent for a normed linear space E: (i) E is an inner-product space or is two-dimensional; (ii) $r_E X = r_X X$ for each bounded convex $X \subset E$. Some related results are obtained, involving the centers of convex sets. (II) Inner-product spaces are characterized by means of sets of points equidistant from two points. (III) Motzkin's characterization of convexity (in terms of existence of unique nearest points) is discussed in an infinite-dimensional setting. (Received March 5, 1958.)

545-22. W. E. Milne and R. R. Reynolds: Stability of a numerical solution of differential equations.

In 1926 Milne published a numerical method for the solution of ordinary differential equations, [Amer. Math. Monthly, vol. 33, pp. 455-460]. This method turns out to be unstable. The purpose of this paper is to show how the occasional application of a quadrature formula using an odd number interval can effectively damp out the unstable oscillation. The method, thus modified, has been tested over a large number of steps and the electronic computer Alwac III-E at Oregon State College with no evidence of instability. (Received March 5, 1958.)

545-23. Kenneth Hoffman and I. M. Singer: On some Banach algebras of Wermer.

Let X be a compact set in the plane with positive Lebesgue measure and no interior. Let A_X denote the Banach algebra of functions continuous on the Riemann sphere S and analytic on S - X. See Wermer [Ann. of Math. vol. 62

(1955) p. 269]. We prove: 1. If X has positive upper density at each point, then the maximal ideal space of A_X is S, and the Silov boundary is X. 2. Let D be a domain in the plane and A_D be the Banach algebra of functions continuous on $\overline{D}$ and analytic on D. Then A_D is not a maximal algebra among the anti-symmetric Banach algebras having $\overline{D}$ as maximal ideal space. (Received March 5, 1958.)

545-24. Jacob Korevaar: Pansions, Fourier transforms and boundary value problems.

In boundary value problems where Fourier transform methods are formally applicable the author's theory of Pansions and Fourier Transforms (Bull. Amer. Math. Soc., Abstracts 63-6-707 through 63-6-712) may be used to obtain more complete results. One example is the classical initial value problem for the heat flow in an infinite rod: $u_{xx} = u_t$, $-\infty < x < \infty$, $0 \leqq t < a$; $u(x,0) = f(x)$. By Fourier transformation one obtains a simple proof of Tychonoff's theorem by which the solution is unique under the condition $u(x,t) = O(\exp cx^2)$. Another example is given by the steady state temperature problem in an infinite strip: $u_{xx} + u_{yy} = 0$, $-\infty < x < \infty$, $0 \leqq y \leqq \pi$; $u(x,0) = f(x), u(x,\pi) = g(x)$. If the solution is required to be $O(\exp c|x|)$ it is unique up to a sum $\sum_{|n| \leqq c} a_n e^{nx} \sin ny$. (Received March 5, 1958.)

545-25. Milton Lees: Approximate solutions of parabolic equations.

A method has been developed for studying the convergence properties of several implicit, finite difference approximations to parabolic partial differential equations of the form $(au_x)_x = F(x,t,u,u_x,u_t)$. Several previously considered finite difference procedures for quasi-linear equations, which were known to converge in the mean square sense, have been shown to actually converge pointwise. Further, a number of difference schemes developed for quasi-linear equations have been applied to nonlinear equations and their pointwise convergence established. Finally, a conjecture of Rose (Quarterly of Applied Math. vol. 14, no. 3, 1956) concerning the mesh ratio restriction for a certain implicit, finite difference approximation to parabolic equations has been answered in the affirmative. (Received March 5, 1958.)

545-26. W. T. Reid: A Prüfer transformation for differential systems.

This paper is concerned with a matrix differential system (*): $Y' = G(x)Z, Z' = -F(x)Y$ in $n \times n$ matrices $Y(x)$, $Z(x)$, where $G(x)$, $F(x)$ are continuous $n \times n$ hermitian matrices on $a \leqq x < \infty$. It is shown that if $Y(x)$, $Z(x)$ is a solution of (*) with $\|Y^*(x)Z^*(x)\|$ of rank n and $Y^*(x)Z(x) - Z^*(x)Y(x) \equiv 0$ then there exists a continuously differentiable nonsingular matrix $R(x)$, and a continuous hermitian matrix $Q(x)$, such that $\Phi' = Q(x)\Psi, \Psi' = -Q(x)\Phi$ has a solution for which $\Phi\Phi^* + \Psi\Psi^* \equiv E$, and $Y(x) = \Phi^*(x)R(x)$, $Z(x) = \Psi^*(x)R(x)$. This result is a direct extension of a theorem of J. H. Barrett (Proc. Amer. Math. Soc. vol. 8 (1957) pp. 510-518) on a Prüfer transformation for real second order matrix differential equations $(P(x)Y')' + F(x)Y = 0$. The procedure of the present paper for the determination of the associated matrix $Q(x)$ is more direct than that employed by Barrett; in particular, the method affords a ready determination of the most general form of $Q(x)$. In addition, it is shown that certain criteria of oscillation and non-oscillation obtained by Barrett may be improved and extended. (Received March 5, 1958.)

545-27. Gert Sabidussi: Graphs with transitively acting groups.

Given a graph X denote by $V(X)$ and $E(X)$ the sets of vertices and edges of X. For each positive integer n and each graph X define nX by $V(nX) = \{(i,x): 1 \leqq i \leqq n, x \in V(X)\}$, $E(nX) = \{[(i,x),(j,y)]: 1 \leqq i \leqq n, 1 \leqq j \leqq n, [x,y] \in E(X)\}$. Theorem: If X is a vertex-transitive graph, then there is a positive integer n such that nX is a color-group. The smallest pos. integer with this property is called the deviation $d(X)$ of X. Very few graphs are known whose deviation is > 1. An example is the Petersen graph P_{10}. Other examples can be obtained as follows: Given a graph X define a dual X^* of X by $V(X^*) = E(X)$, $E(X^*) = \{[e,e']: e,e' \in E(X)$, e and e' adjacent in $X\}$. If p is a prime of the form $4n + 3$ then $d(C(p)^*) > 1$, and tends to infinity with p. If X is edge-transitive, then X^* is vertex-transitive, andthe notion of deviation can thus be extended to edge-transitive graphs. Using this notion of dual it is possible to determine the structure of the following types of graphs: Strictly edge-transitive graphs, transitive graphs with involutorial edges, and transitive graphs without involutorial edges. (Received March 5, 1958.)

545-28. F. L. Spitzer: Toeplitz matrices and the absorption problem.

The two-sided absorption problem for sums of identically distributed independent symmetric lattice random variables is found to be connected with the theory of orthogonal polynomials defined on the unit circle. Let X_i be the random variables, $\Pr(X_i = \pm k) = c_k = c_{-k}$, $f(\theta) = \sum C_k e^{ik\theta}$ their characteristic function, $S_k = X_1 + \ldots + X_k$ their partial sums. Let $P_{ke}^{\nu}(N)$ be the probability that $S_\nu + k = e$ and that $0 \leqq S_i + k \leqq N$ for $i = 1, \ldots, \nu$. For $0 \leqq \lambda \leqq 1$, the following representation formula is then valid: $\sum_{\nu=0}^{\infty} \sum_{\ell=0}^{N} \sum_{k=0}^{N} P_{ke}^{\nu}(N) \lambda^{\nu} t^k S^e = \sum_{k=0}^{N} P_k(t;\lambda) P_k(S;\lambda)$, where $P_k(t;\lambda)$ form a sequence of polynomials of degree k in t. They are uniquely determined by requiring that the coefficient of the highest power be positive and that they be orthonormal on the unit circle, viz. $(2\pi)^{-1} \int_0^{2\pi} [1 - f(\theta)] P_k(e^{i\theta};\lambda) P_e(e^{i\theta};\lambda) d\theta = \delta_{ke}$. G. Szegö has shown what happens as $N \rightarrow \infty$ in our representation formula. Probabilistically, this passage to the limit solves the one-sided absorption problem. (Received March 5, 1958.)

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546-1. Herbert Federer: *A general integralgeometric formula, with application to curvatures*. Preliminary report.

Suppose A and B are Borel sets contained in p and q dimensional smooth submanifolds of Euclidean n space, let $r = p + q - n$, and consider the kinematic integral $\int_G \int_{A \cap g(B)} u(x,g)\, dH^r x\, dmg$ where G is the group of isometries of n space with the Haar measure m and H^r is r dimensional Hausdorff measure. Assuming that $u(x,g)$ is determined by $x \in A$, $g^{-1}(x) \in B$ and the restriction of g^{-1} to the tangent plane of $A \cap g(B)$ at x, it is shown that the kinematic integral equals $\int_A \int_B v(x,y) dH^p x dH^q y$ where v is computable from u by integration over the p and q dimensional orthogonal groups. This general result applies for instance to the even dimensional curvature integrals k_e defined by Hermann Weyl (*On the volume of tubes*, Amer. J. Math. vol. 61 (1939)). It is shown that $\int_G k_e[A \cap g(B)]\, dmg = \sum_{i=0}^{e/2} c_{2i} k_{2i}(A) k_{e-2i}(B)$ where $c_0, c_2, \ldots, c_e$ depend only on p, q, n. Other applications are contemplated, in particular to odd dimensional curvatures of manifolds with boundary and to Pontrjagin numbers. (Received January 13, 1958.)

546-2. Shreeram Abhyankar: *Local fundamental groups of Kroneckerian varieties*.

The local fundamental group at the origin in the complex x,y plane with the x and y axes removed is free abelian with two generators. The author had algebraized this phenomenon and carried it over to abstract algebraic varieties of any dimension (Amer. J. Math. vol. 77 (1955)) and the proof there used the "Purity of the branch locus". Now this local theory of normal crossings is extended to algebroid and Kroneckerian (i.e. Absolute) geometry. Here Jacobian theory is not available and the proofs are obtained first for the "surface" case using: (1) the M-adic divisor is a projective line, (2) that a nonabelian tame extension of the projective line must have at least three branch points, (3) connectedness theorem in the general form proved by Chow (forthcoming), and (4) a theorem on limits of sequences of quadratic transforms of two dimensional regular local rings. Higher dimensional case is reduced to the surface case in two ways; *inner reduction*: by a "local Bertini theorem" conjectured by the author and proved by Chow (forthcoming), *outer reduction*: will be described in a later paper. In this set up "Purity" is an incidental corollary. (Received January 10, 1958.)

546-3. Jerome Blackman and Harry Pollard: *The finite convolution transform*.

Consider the convolution transform $f(x) = \int_0^x \phi(x - t)dk(x)$. The classical solution for ϕ, given f and k, involves the Laplace transform and requires restrictive assumptions about the behavior of ϕ and k at infinity. Further, a consideration of the equation indicates that a knowledge of f(x) in $0 \leqq x < a$ should be sufficient to determine ϕ in $0 \leqq x < a$ if k(x) is assumed to have positive variation in every neighborhood of the origin. The authors have solved the equation in the interval $0 \leqq x < a$ under the assumptions that k is of bounded variation and ϕ is Borel measurable in the interval. The methods used are variations of the operator techniques which have been used on convolution transforms in the past few years. (Received February 5, 1958.)

546-4. A. R. Amir-Moéz: *Some equalities in a unitary space leading to equalities concerning singular values of sets of matrices*.

If $\{\alpha_1,\ldots,\alpha_k\}$ is a set of orthogonal vectors, $\{a_1,\ldots,a_k\}$ a set of positive numbers for which $\sum_{i=1}^k a_i = 1$, and $\xi = \sum_{i=1}^k a_i \alpha_i$, then $(\xi, \alpha_i - \alpha_j) = 0$, $i, j = 1,\ldots,k$ if and only if $1/|\xi|^2 = \sum_{i=1}^k 1/|\alpha_i|^2$, and $(\xi, \alpha_i/|\alpha_i|) = (\xi, \alpha_j/|\alpha_j|)$ implies $k^{1/2}/|\xi| = \sum_{i=1}^k 1/|\alpha_i|$. This theorem leads to a similar relation among the singular values of a set of orthogonal matrices in the sense that $(A,B) = \sum a_{ij} \bar{b}_{ij}$. Independently of the norm and inner product, if $\{A_1,\ldots,A_k\}$, $k < n^2$, is a set of matrices for which $A_i^* A_j = 0$, $i < j$, a_i's as above, and if $B = \sum_{i=1}^k a_i A_i$, $\lambda_1 \geqq \ldots \geqq \lambda_n$ are the singular values of B and $\psi_{i1} \geqq \ldots \geqq \psi_{in}$ the singular values of A_i, then $B^*([1/\psi_{ip}]A_i - [1/\psi_{jp}]A_j) = 0$ implies $k^{1/2}/\lambda_p = \sum_{i=1}^k 1/\psi_{ip}$ and $B^*(A_i - A_j) = 0$ implies $1/\lambda_p^2 = \sum_{i=1}^k 1/\psi_{ip}^2$, $p = 1,\ldots,n$. (Received February 6, 1958.)

546-5. F. A. Homann: *On the osculating spaces to analytic projective curves*.

Let C: $x(t) = \{x^1(t),\ldots,x^{n+1}(t)\}$ be for $-\infty < t < \infty$ a proper analytic curve immersed in the real n-dimensional projective space S_n. Let $P_1(x)$, $P_2(x)$, $\ldots, P_n(x)$ be the first n consecutive hyperplanes to C at the point x(t), and let them be assigned coordinates in terms of their defining equations. Then it is shown that the osculating space S_p $(0 \leqq p \leqq n - 1)$ at $x(t)$ determined by the $p + 1$ consecutive derivative points $x, dx/dt, \ldots, d^px/dt^p$ coincides with the space determined by the intersection of $P_1(x), P_2(x), \ldots, P_{n-p}(x)$. Consequently S_p may be assigned Grassmann coordinates formed from the derivative point coordinates and dual Grassmann coordinates formed from the consecutive hyperplane coordinates, and the explicit relationship between the dual coordinate systems is calculated by Laplace's determinantal expansion and the wronskian determinant of the $x^i(t)$. Also, the case of self-adjoint curves is discussed briefly. (Received January 15, 1958.)

546-6. Costas Kassimatis: *Functions which have generalized Riemann derivatives*.

The definitions of the generalized Riemann derivative and the de La Vallée Poussin derivative can be found in : J. Marcinkiewicz and A. Zygmund, *On the differentiability of functions and summability of trigonometrical series*, Fund. Math. vol. 26 (1936) pp. 1-43. The object of the present paper is to study the relations between two functions $F(x)$ and $G(x)$ where the nth generalized Riemann derivative of the continuous function $F(x) - G(x)$ is equal to zero. By defining a class K_n of continuous functions and by generalizing the methods of Denjoy (A. Denjoy, *Leçons sur le calcul des coefficients d'une série trigonométrique*, Paris, 1941 and 1949, pp. 18-19) the following theorem is proved: *If the function* $f(x)$ *is continuous on* $[a,b]$ *and belongs to the class* K_n *then the* nth *divided difference of* $f(x)$ *relative to* $n + 1$ *arbitrary distinct points of* $[a,b]$ *lies in the interval* $[(1/n!) \inf \underline{D}^n f(x), (1/n!) \sup \overline{D}^n f(x)]$, $a < x < b$. (Received February 10, 1958.)

546-7. Abolghassem Ghaffari: *On the asymptotic behavior of the integral curves of a certain nonlinear differential equation*.

It is the purpose of this paper to investigate the behavior in the large of the totality of the paths of a dynamical system defined by the nonlinear differential equation (1) $dy/dz = [y(x^2 + y^2 - 2x - 3)(x^2 + y^2 - 2x - 8) + x] \cdot [x(x^2 + y^2 - 2x - 3)(x^2 + y^2 - 2x - 8) - y]^{-1}$ in the phase-plane. From general theory one can obtain the following results: *The origin* is the only elementary critical point of (1), which is an *unstable focus*; the *equator* is a *limit-cycle*. In polar form (1) transforms into (2) $d\rho/d\theta = \rho(\rho^2 - 2\rho\cos\theta - 3)(\rho^2 - 2\rho\cos\theta - 8)$, which is partially discussed by Poincaré (Oeuvres, Tome I. p. 83). One can observe that the origin and the circles $\rho^2 - 2\rho\cos\theta - 3 = 0$, $\rho^2 - 2\rho\cos\theta - 8 = 0$ *are curves with contact while the circles without contact are defined by* $0 < \rho < 1$ and $\rho > 4$. It is shown that the circular region $1 < \rho < 4$ contains *only the two limit-cycles C_1 and C_2, so that the totality of the paths of (1) con*sists, besides the equator and the two limit-cycles C_1 and C_2, of a first family of spirals (inside C_1) spiralling away from the origin, a second family of spirals (between C_1 and C_2) spiralling away from C_1, and finally a third family of spirals (between C_2 and the equator) approaching asymptotically the limit-cycle C_2 and the equator. (Received February 14, 1958.)

546-8. L. W. Small: A connection between many valued logics and the two-valued system.

A three valued logic is generalized as a transformation of the usual two valued system. Given the two valued system Ti ($i = 1, 2, \ldots, n$). By means of transformations relating to the position of the T elements we obtain the generalization to a three valued logic. For instance if we wish to change from our original logic a system of steps is added between the true value of an element and its negate, i.e., the number of steps between c and $\bar{c}$ exceeds one. This number may then assume any positive number greater than one. A different system is calculated for the "and" and "or" relations. Thus we have a system of many valued logics dependent upon the simple two value logic. The generalization is further expanded by building truth function tables that are related to our standard in different positions. (Received February 13, 1958.)

546-9. R. K. Getoor: Markov operators and their associated semi-groups.

Let $x(t)$ be a temporally homogeneous Markov process in E^N having right continuous paths and such that the associated semi-group $(T_t\phi)(x) = \int\phi(y)p(t,x,dy)$ takes bounded continuous functions into bounded continuous functions, where $p(t,x,A)$ is the transition probability function of $x(t)$. We assume further that $p(t,x,A)$ has a density $f(t,x,y)$ with respect to a Radon measure, m, and f satisfies $\int f(t,x,y)dm(x) < Ke^{\alpha t}$. Under these conditions it is shown that $\{T_t : t \geqq 0\}$ is a strongly continuous ($t \geqq 0$) semi-group of bounded operators on $L_2(m)$. Let Ω be the infinitesimal generator of $\{T_t : t \geqq 0\}$, we then define in a natural manner the "generalized restriction", Ω_G, of Ω to an open subset, G, of E^N and obtain the Green's function of the Dirichlet problem for Ω_G.

We show that if Ω is a local operator then Ω_G is the ordinary restriction of Ω to G and finally we obtain some results on the asymptotic behavior of the spectrum of Ω_G. (Received February 19, 1958.)

546-10. Seymour Schuster: Pencils of null systems. II.

A second (for first, see Abstract 538-57) synthetic definition is given for a pencil of null systems: Let PQRST be a self-polar skew pentagon in complex projective space of three dimensions, with P, Q, R and S fixed, while T varies on a line t. It is stipulated that line t be in the fixed polar plane of point T. Under this definition of a pencil of null systems, there are three types of pencils according as t meets none, one or two of the fixed edges of PQRST. These are called the general, parabolic and degenerate systems, respectively. The product of two null polarities of the same general system is a general axial homography, while the product of two null polarities of the same parabolic system is a biaxial homography. Conversely, every general axial homography can be expressed as the product of two null polarities of the same general system, and every biaxial homography can be expressed as the product of two null polarities of the same parabolic system. The self-polar lines of each polarity form a linear complex. Thus, each pencil of null systems leads to a pencil of linear complexes, with the lines common to all complexes of the pencil forming a linear congruence. In this manner the general, parabolic and degenerate systems yield general, parabolic and degenerate linear congruences, respectively. (Received February 19, 1958.)

546-11. B. A. Fleishman: Progressing waves in an infinite nonlinear string. Preliminary report.

Ficken and Fleishman (Comm. Pure Appl. Math. vol. 10 (1957) pp. 331-356) have demonstrated the existence of time-periodic solutions to the nonlinear wave equation $u_{tt} - u_{xx} = -\alpha u - 2\kappa u_t - \beta u^3 + b(x,t)$, where $\kappa > 0$, $\alpha > 0$, and $|\beta|$ is small. It is proved here that the associated equation (1) $u_{tt} - u_{xx} = -\alpha u - \beta u^3$ has progressing-wave solutions, of different types. Suppose $\alpha > 0$. Let v be any real number such that $v^2 > 1$. Then if $\beta > 0$, elliptic functions of the argument $s = x - vt$ are solutions of (1). There is no restriction here on the magnitude of β; also, these solutions are, incidentally, periodic in t. If $\beta < 0$, at least two types of waves exist: (a) for small $|\beta|$, small-amplitude periodic waves similar to those occurring when $\beta > 0$; (b) for arbitrary β, a class of nonperiodic solutions. The method used is similar to that used by van der Pol (Indag. Math. vol. 19 (1957) pp. 477-480) to treat another nonlinear wave equation. (Received February 21, 1958.)

546-12. O. H. Hamilton: Fixed points for special transformations of plane continua.

Let M be a bounded plane continuum and T a continuous transformation of M into itself. Let T_1 be the transformation of M into the unit circle S such that for each point z in M, $T_1(z)$ is the point on S whose direction from O is the same as the direction from z to T(z). Assume T_1 to be an inessential mapping of M into S. Then (1) if T^2 leaves exactly two points of M fixed, T leaves at least one point of M fixed; (2) if T is of period 3, T leaves a point of M fixed. (Received February 24, 1958.)

546-13. D. L. Slotnick: Asymptotic behavior of solutions of canonical systems near a closed, unstable orbit.

An investigation is made of a class of canonical systems with Hamiltonian $H(x,y,t) = -\omega(x^2 + y^2) + H_s(x,y,t) + H_{s+1}(x,y,t) + \ldots$, a convergent power series in some neighborhood of the origin. $H_n(x,y,t)$, where $n \geq s \geq 3$, is a homogeneous polynomial of degree n in the real variables x and y with coefficients continuous functions of t having period 2π. ω is a positive constant. The stability question for these systems is still unanswered. Moser has proved [Nachr. Gess. Wiss. Göttingen vol. 6] that for certain ω, the definiteness of H_s implies a strong growth condition on the solutions. The unstable systems dealt with here are defined by an appropriate indefiniteness condition on H_s. Following Poincaré, the mapping of the initial values into the values of the solutions corresponding to these initial values, at the time 2π, is introduced. For this mapping the existence of a unique invariant curve is proved. It is moreover proved that this curve, which corresponds to a one-parameter family of unstable solutions, is analytic away from the origin and is infinitely differentiable at the origin. An example is given demonstrating that this curve may have an essential singularity at the origin. (Received February 24, 1958.)

546-14. J. C. Scanlon: Poincaré's perturbation method and topological degree.

If Poincaré's perturbation method is used to investigate the periodic solutions of a system $(^*)x = Ax + f(x,t,\epsilon)$, the problem is reduced to that of solving the bifurcation system, n equations in n unknowns, where n is the degree of degeneracy, i.e., the number of linearly independent solutions of the variational equation. If the form of the bifurcation system derived by Coddington and Levinson (Contributions to the theory of nonlinear oscillations, vol.II) is used and if certain conditions are imposed on $f(x,t,\epsilon)$, it can be shown that the topological degree corresponding to the bifurcation system is nonzero, and thus periodic solutions of (*) exist. Both autonomous and nonautonomous systems can be treated, and the number of distinct solutions can be determined in some cases. (Received February 25, 1958.)

546-15. Chandler Davis: The set of nonlinearity of a convex piecewise-linear function.

Let D be a graph dividing the plane into a finite number of convex regions A; let D be cubic (3 edges on each vertex); let no two incident edges be collinear. Call D realizable in case there exists convex real-valued f on the plane, linear except on D. Definition: A_{i-1} A_i, A_{i+1} being successive vertices of region A, and l being the edge on A_i outside A, then the slope of l in the coordinate system with origin A_i and unit points A_{i+1} and A_{i-1} is the i-th intrinsic slope of A. Theorem: D is realizable if and only if, for each bounded A, the product of the intrinsic slopes of A is 1; f is then essentially unique. Qualitative features of realizable D are deduced. (Received February 27, 1958.)

546-16. A. M. Garsia: Calculation of conformal parameters of imbedded Riemann surfaces.

A method for the calculation of conformal parameters of imbedded Riemann surfaces, is introduced. The method is based upon the fact that, apart from some topological restrictions, every imbedded Riemann surface can be considered as an envelope of Riemann surfaces of genus zero. It is shown when and how a given Riemann surface can be conformally mapped upon the enveloping surfaces. Such mappings then generate a representation of the given Riemann surface upon a domain of the unit sphere bounded by circular boundaries, identified in couples by moebius transformations. The method applied to "canal surfaces" of 3 dimensional space leads to a simple characterization of their conformal parameters. For such surfaces the conformal parameters are closely related to the invariants of moebius transformations of 3-space generated by the enveloping spheres. (Received February 27, 1958.)

546-17. R. P. Gosselin: A theorem on L^p Fourier series.

Given a sequence $\{m_\nu\}$ of distinct positive integers, let $\sigma(n)$ be the number of terms of the sequence not exceeding n. We say $\{m_\nu\}$ satisfies (C_p) if $\limsup (\log n)^\alpha \sigma(n)/n \geq 1$ where $\alpha = (2 - p)/2(p - 1)$. Given f in L^p, $1 < p \leqq 2$, there is a sequence $\{m_\nu\}$ satisfying (C_p) such that $\lim s_{m_\nu}(x;f) = f$ almost everywhere, where $s_m(x;f)$ denotes the mth partial sum of the Fourier series of f at x. This generalizes a previous result concerning only the L^2 case (Proc. Amer. Math. Soc. vol.7 (1956) pp. 392-397). The proof involves a theorem of Littlewood and Paley (Proc. London Math. Soc. vol. 42 (1937) pp. 52-89). (Received February 27, 1958.)

546-18. M. S. Klamkin: On the steady-state solution of a heat conduction problem with Stephan-Boltzmann radiation losses.

Physically, the problem is to determine the heat flux into a long thin rod by means of thermocouples. The complication here is that the rod becomes very hot near the heat input end that we have to account for the Stephan-Boltzmann radiation losses (i.e., $Q_r = \epsilon A(T^4 - T_m^4)$). For the case of constant thermal properties, we obtain the exact steady-state solution in terms of an integral which can be expressed explicitly in terms of the Weierstrassian elliptic function and its associated Zeta and Sigma functions. For convenience we have tabulated an associated function $\psi(x) = \int_x^\infty dx/(x^5 - 5x + 4)^{1/2}$ and also give its expansions for large and small (x - 1). For the case of variable thermal properties we obtain the solution in terms of a quadrature. (Received February 27, 1958).

546-19. H. W. E. Schwerdtfeger: On the method of minimum deviation.

In their paper, Mat. Sbornik vol. 31 (1952) p. 315, Krasnos'el'skii and Krein proposed the following method for the solution of a linear system $Ax = b$ with a real positive symmetric $n \times n$-matrix A: Let $x^{(0)}$ be an arbitrary initial vector and $d = Ax^{(0)} - b$ the corresponding deviation. As first approximation to the solution x choose $x^{(1)} = x^{(0)} + \gamma d$ and determine γ such that the deviation vector $d^{(1)} = Ax^{(1)} - b = d + \gamma \cdot Ad$ has minimum length. This method, although theoretically interesting, proves to be poor from the numerical point of view. It can, however, be improved by substituting instead of the number factor γ a diagonal matrix $C = [c_1\ c_2 \ldots c_n]$. In order to reduce the number of unknowns linear relations of the form $c_1 = c_2 = \ldots = c_\ell = \gamma_1, \ldots, c_{m+1} = \ldots = c_n = \gamma_k$ may be assumed. Then a quadratic function $f_k(\gamma_1, \ldots, \gamma_k)$ is to be minimized whereas $f(\gamma) = f_k(\gamma, \ldots, \gamma)$ is the function to be minimized in the original process. The choice of the γ_i can be adapted to numerical conditions. (Received February 27, 1958.)

546-20. Günter Lumer and Marvin Rosenblum: Linear operator equations.

$\mathcal{A}$ being a complex Banach algebra with unit, let $\mathcal{B}$ denote the Banach algebra of operators on $\mathcal{A}$ considered as a Banach space. For $u, v \in \mathcal{A}$ define $T \in \mathcal{B}$ by $Tx = ux - xv$. This operator was studied by M. Rosenblum (Duke Math. J. vol. 23 (1956)). Here it is shown that the results of this paper can be extended to the case of all operators of the form $Sx = \sum_d u_d x v_d$ (called "elementary operators"), under the hypothesis that the $\{u_d\}$ and $\{v_d\}$ form (separately) commutative subsets of $\mathcal{A}$. Applications are made to the solution of linear equations in (not necessarily commutative) Banach algebras (elementary operators playing the role that the simple coefficients play with commutative algebras). For example, the usual sufficient condition for the existence of a (unique) solution of a system of n equations with n unknowns, is generalized (in terms of a determinant involving the spectra of the coefficients in $\mathcal{A}$). (Received February 28, 1958.)

546-21. T. G. Room: Spinors and quaternions:

Assume a matrix representation in $N = 2^n$ dimensions of a Clifford algebra with $2n + 1$ units, in which $n + 1$ of the matrices, p_i, $i = 0, \ldots, n$, are symmetrical and the remaining n matrices, q_j, $j = 1, \ldots, n$, are skew. Call spinor any point u of projective space of $N - 1$ dimensions which is such that the $2n + 2$ points u, $p_i u$, $q_j u$ span a space of at most n projective dimensions. Then the matrix A whose elements are $(u^T u)^{-1} (u^T q_j p_i u)$—first row $(u^T u)^{-1} (u^T p_i u)$—is such that $AA^T = \underline{1}$, $\det A = 1$, and all its minors are of the form $(u^T u)^{-1} (u^T \prod p_i \prod q_j u)$. Readily calculable explicit forms can be given for all these expressions. By a simple interchange among elements and minors of A, a matrix may be obtained which is orthogonal in relation to a quadratic form of any given signature. There is a one-to-one correspondence between spinors and the members of O^+_{n+1}.

Using the components of the spinor u as coefficients of a matrix linear function, we derive from u a unique matrix U (the *quaternion*) such that if A, B of 0^+_{n+1} correspond to spinors u,v, and U,V are the corresponding quaternions, then Uv is the spinor and UV the quaternion corresponding to AB. There is also a fixed matrix J such that Ju is the spinor, while U^T is the quaternion, corresponding to A^T. The connection with the classical theory of spinors may be established by way of the maximum isotropic subspaces (generating spaces) of the quadratic form $\sum_0^n(y_i^2 - z_i^2)$ in vector space of 2n + 2 dimensions, these subspaces being given by sets of equations y = Az. (Received February 28, 1958.)

546-22. R. H. Bing: *The cartesian product of a certain nonmanifold and a line is* E^4.

The paper *A decomposition of* E^3 *into points and tame arcs such that the decomposition space is topologically different from* E^3, Ann. of Math. vol. 65 (1957) pp. 484-500, describes a decomposition space B with properties as suggested in the title of the paper. We now show that the cartesian product of a line E^1 and this space B is topologically E^4. The noncompactness of B and E^1 is not a critical issue since the cartesian product of a circle and the one point compactification of B is toplogically $S^1 \times S^3$. (Received February 28, 1958.)

546-23. Edward Nelson: *Measurability of stochastic processes*.
WITHDRAWN.

Let T and X be [0,1] (for simplicity), Ω the space of all functions from T to X, in the product topology. The following are Borel sets in Ω: all continuous except for $\{$no exceptions; sets of measure 0; sets of first category; jumps$\}$. Let Pr be a regular Borel probability measure on Ω such that $\xi(s,\cdot) \rightarrow \xi(t,\cdot)$ in measure as $s \rightarrow t$ (where $\xi(t,\omega) = \omega(t)$). Then $\xi(t,\omega)$ is jointly measurable, for any measure on T. The set O(Pr) of fixed points of discontinuity is of the first category in T, and almost all ω are Riemann integrable if and only if O(Pr) has zero Lebesgue measure. The existence and uniqueness of Pr follows at once from the Riesz-Markoff and Stone-Weierstrass theorems, given the finite joint distributions. The latter determine all values of Pr in principle. The above results are obtained by carrying out this reduction in practice. Borel measures on Ω are due to Kakutani (see Doob, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 15-30). (Received February 28, 1958.)

546-24. Smbat Abian and A. B. Brown: *A note on the solution of the differential equation of the type* $g(x,y,y') = 0$.

An adaptation of Picard's method of successive approximations is used to prove the existence and uniqueness of the solution of an implicit differential equation of the type $g(x,y,y') = 0$ with the initial condition (a,b), under the hypotheses that g is a real valued continuous function defined in a neighborhood $N \subset E^3$ of the point (a,b,c) with $g(a,b,c) = 0$, and that there are three positive numbers H, C, D such that, for (x,y,z) in N, $|g(x,y_2,z) - g(x,y_1,z)| \leqq H|y_2 - y_1|$ and, if $z_1 \neq z_2$, $C \leqq (g(x,y,z_2) - g(x,y,z_1))/(z_2 - z_1) \leqq D$. In the paper a specific algorithm is given for finding the solution by a succession of quadratures, without solving the given equation $g(x,y,y') = 0$ explicitly for y'. (Received February 3, 1958.)

546-25. Marvin Rosenblum: *On the Hilbert matrix*. II.

The Hilbert matrix $H_k = ((m + n + 1 - k))^{-1}$ $m,n = 0,1,\ldots$ is studied as an operator on ℓ^2. Its spectrum and eigenfunctions are determined and the nonhomogeneous equation $\lambda x - H_k x = g \in \ell^2$ is solved when λ is not in the spectrum of H_k. The method of analysis is to exhibit an isomorphism U of ℓ^2 onto $L^2(0,\infty)$ and a Sturm-Liouville operator L_k such that $UH_kU^{-1} = \pi \operatorname{sech} \pi L_k^{1/2}$. L_k is then diagonalized by the Titchmarsh-Kodaira procedure and these results interpreted in terms of H_k. (Received March 3, 1958.)

546-26. E. P. Miles, Jr.: *The Cauchy problem for the iterated wave equation*.

Let $\Box u = \sum_{i=1}^{k} u_{x_i x_i} - u_{tt}$. Consider the Cauchy problem: (A) $\Box^m u = 0$; (B) $\partial^p u/\partial t^p(x_1,x_2,\ldots x_k,0) = f_p$, $p = 0,1,\ldots,2m - 1$. From a basic set for (A) originally developed in collaboration with E. Williams the author develops a new basic set associated in the most natural way with the problem (A); (B). The generic element of this new set $P^{n,m}_{a_1,\ldots,a_k,p}$ contains only one term of degree less than $2m$ in t, viz., $x_1^{a_1}\ldots x_k^{a_k} t^p/p!$ With polynomial data $f_p = \sum_{q=0}^{D(p)} \sum_{\Sigma a_i = q} A^{q,p}_{a_1,\ldots a_k} x_1^{a_1}\ldots x_k^{a_k}$ the problem (A); (B) has solution $u = \sum_{p=0}^{2m-1} F_p$ where $F_p = \sum\sum A^{q,p}_{a_1,\ldots,a_k} P^{q+p,m}_{a_1,\ldots,a_k,p}$. When the f_p are polyharmonic functions in a region R of the x-hyperplane the author obtains explicit solutions of (A); (B) analytic in the infinite right cylindrical region in (x,t) space with base R, and hence across the characteristic boundary of the retrograde cone. This provides an illustration of singularities in the data not being propagated along the characteristics. (Received March 3, 1958.)

546-27. A. E. Danese: Some identities and inequalities involving ultraspherical polynomials.

A positive representation of $\Omega_n^\lambda(x) = p_{n+1}^\lambda(x)p_{n+2}^\lambda(x) - p_n^\lambda(x)p_{n+3}^\lambda(x)$, where $p_n^\lambda(x) = P_n^\lambda(x)/P_n^\lambda(1)$, $P_n^\lambda(x)$ the ultraspherical polynomial of degree n and order λ, is obtained, from which follows for $\lambda > 0$ that $\Omega_n^\lambda(x) \geqq 0$ for $0 \leqq x \leqq 1$ and $x \leqq -1$, with equality for $x = -1,0,1$ only and $\Omega_n^\lambda(x) < 0$ for $-1 < x < 0$ and $x > 1$. Sharper estimates of these inequalities are also obtained. A relationship between $\Omega_n^\lambda(x)$ and the corresponding expression with the derivatives of the ultraspherical polynomials is established. Relationships between $\Omega_n^\lambda(x)$ and $d/dx\, \Delta_n^\lambda(x)$ and $d^2/dx^2\, \Delta_n^\lambda(x)$, where $\Delta_n^\lambda(x) = [p_n^\lambda(x)]^2 - p_{n+1}^\lambda(x)p_{n-1}^\lambda(x)$, are obtained. Some inequalities are obtained for $[p_n^{\alpha,\beta}(x)]^2 - p_{n+1}^{\alpha,\beta}(x)p_{n-1}^{\alpha,\beta}(x)$ where $p_n^{\alpha,\beta}(x)$ is the normalized Jacobi polynomial. The polynomials $[p_{2n+1}^\lambda(x)]^2 - p_{2n-1}^\lambda(x)p_{2n+3}^\lambda(x)$ are also studied. (Received March 3, 1958.)

546-28. Charles Fox: Fractional integration and chain transforms.

In Proc. Amer. Math. Soc. (1954) a chain transform is defined as the double set of equations (1) $g_{p+1}(x) = \int_0^\infty r_p(x/u)g_p(u)du/u$, (2) $\int_0^\infty \ell_q(x/u)g_{q+1}(u)du/u = g_q(u)$, where (i) the p's and the q's ($p \neq q$) between them run through the first n positive integers, (ii) $g_{n+1}(u) = g_1(u)$ and (iii) each function $g_i(u)$, $(i = 1,2\ldots,n)$, occurs twice in the double set (1),(2), once on the right hand side and once on the left. When $n = 2$ the system reduces to the Generalized Fourier Transform. When these conditions are satisfied the n functions $r_p(x)$ and $\ell_q(x)$ are called the kernels of a chain transform. It is now proved that the property of forming the n kernels of a chain transform is widely invariant after the operations of fractional integration and differentiation have been applied to these kernels. Consequently from one such set of kernels we can, by means of fractional integration and differentiation, derive an infinite number of other such sets. (Received March 3, 1958.)

546-29. V. E. Beneš: Characterization and decomposition of stochastic processes with stationary independent increments.

Let x_t be a centered stochastic process with stationary independent increments. We say that x_t is a compound Poisson process if $\log E \exp\{i\mu(x_t - x_s)\}$

has the form $c(t - s)[\phi(\mu)-1]$, $c \geqq 0$, ϕ a c.f. The following main results are proved: (1) $x_t = y_t + \sum_p z_p(t) + bt$, where y_t is a Brownian motion, b is a constant, and the z_p are independent (of each other and of y_t) compound Poisson processes, all (except possibly one) centered at expectations, the series converging a.s.: (2) x_t is a compound Poisson process if and only if, with probability one, the sample functions are step-functions with a finite number of jumps in any finite interval. (Received March 3, 1958.)

546-30. F. R. Fitch: *Representation of sequential circuits in combinatory logic*.

A consistent logic L is presented within which every sequential circuit, viewed as a transformation of sequences into sequences, is definable. L has H. B. Curry's theory of combinators as a subsystem. Operators B_n, C_n, W, and Z_n are defined such that $B_n fga_1 \ldots a_n = f(ga_1 \ldots a_n)$, $C_n fa_1 \ldots a_{n-1}a_n a_{n+1} = fa_1 \ldots a_{n-1}a_{n+1}a_n$, $Wfa = faa$, and $Z_n fa_2 \ldots a_n = f(Z_n fa_2 \ldots a_n)a_2 \ldots a_n$. Boolean and delay circuit elements are represented in L by operators &, V, ~, and D. If f and g respectively represent in L the n-input and m-input circuits F and G, then $B_m fg$ represents the circuit obtained by feeding the output of G into the first input of F, and $C_k f$ represents the result of interchanging the kth and k + 1st inputs of F, and Wf represents the result of joining the first two inputs of F to form a single input, and $Z_n f$ represents the result of feeding a branch from the output of F into the first input of F itself. Several apparently novel feedback equations are stated, two of which are: $Z_{n-1}(Z_n f) = Z_{n-1}(Wf)$, and $W(Z_n f) = Z_{n-1}(C_1(W(C_2(C_1 f))))$. (Received March 3, 1958.)

546-31. G. D. Findlay and Joachim Lambek: *A generalized ring of quotients*. II.

The maximal ring Q of right quotients of an arbitrary ring R, due to Utumi and the present authors (see Abstract 538-71), is constructed in several equivalent ways. Two special cases are considered: If R is a ring with unity and minimal condition, Q is the ring of left-endomorphisms of its smallest two-sided ideal whose left-annihilator is zero. If R is a commutative ring with unity, Q is also commutative but may be larger than the usual full ring of quotients consisting of the ratios $r'r^{-1}$ where $r', r \in R$ and the annihilator of r is zero. (Received March 3, 1958.)

546-32. J. W. Gray: Contact manifolds.

A manifold M^{2n+1} is contact if there exist coordinate transformations which preserve the form $dz - \sum_{i=1}^{n} y^i dx^i$ in E^{2n+1} (with coordinates $x^1,\dots,x^n$, $y^1,\dots,y^n,z$) modulo a nonzero factor τ_{ij}. The 1-cocycle τ_{ij} represents the first Stiefel-Whitney class of M in a natural manner, and hence there exists a globally defined 1-form α of maximal rank on a contact manifold M if and only if M is orientable. Such a 1-form α exists if and only if M^{2n+1} is the boundary of a manifold N^{2n+2} which carries a 1-form β such that $(d\beta)^{n+1} \neq 0$ and $\beta | M^{2n+1} = \alpha$. A manifold M^{2n+1} is almost-contact if there exists a reduction of the structure group of the tangent bundle to $1 \times U(n)$. A contact manifold is almost contact. The primary obstruction to an almost-contact structure is the third Stiefel-Whitney class, w_3. It follows that M^5 admits a U(2)-structure if and only if $w_3(M^5) = 0$. (Received March 3, 1958.)

546-33. H. S. Shapiro: Linear extremal problems for polynomials.

Let π_n denote the linear space of polynomials of degree n with complex coefficients: $f(z) = \sum_{k=0}^{n} a_k z^k$, where z denotes a complex variable, normed by $\|f\| = \max_{|z| \leq 1} |f(z)|$. The most general linear functional on π_n is, T: $f \to t_0 a_0 + \dots + t_n a_n$, where the t_i are complex numbers characterising T. The following problems are studied: (i) Characterise $\|T\|$, the maximum of $|Tf|$ over all f of norm 1, directly in terms of $t_0,\dots,t_n$. (ii) Characterise the extremal polynomials, that is, those $f \in \pi^n$ of norm 1 for which, $Tf = \|T\|$. Certain general results are obtained, by relating the given problem to a "dual" extremal moment problem for functions of bounded variation. In particular, the author explicitly characterises all T which admit one of the monomials z^k $(0 \leqq k \leqq n)$ as extremal, and proves that in the remaining cases T admits a unique representation, $Tf \equiv \sum_{j=1}^{r} c_j f(\omega_j)$, where $1 \leqq r \leqq n$, the ω_j are numbers of modulus 1, and the c_j are complex numbers satisfying $\sum |c_j| \equiv \|T\|$. Uniqueness of the extremal polynomial and related questions are also studied, and the results applied to several special functionals, notably $Tf = f'(\alpha)$ and $Tf = a_0 + ca_1$ $(c \neq 0)$. (Received March 3, 1958.)

546-34. Donald Bratton: New results in the theory and techniques of Chebyshev fitting.

Let f be an element and V a finite dimensional vectorial subspace of the space $\mathcal{C}(D)$ of the continuous functions on a compact space D. There exists a set S of n + 1 points of D (n being the dimension of V) such that each best fit to f out of V uniformly over D is a best fit to f out of V over S. S is called the set of critical points of f. When D is a compact interval and V the set of polynomials of degree $< n$, the best fit to f at n + 1 arbitrary points (x_i) $(0 \leqq i \leqq n)$ of D is at the distance $\rho = (\sum_i \prod_{j \neq i} |x_i - x_j|^{-1})^{-1} \; |f(x_0, \ldots, x_n)|$, where $f(x_0, \ldots, x_n)$ is Newton's divided difference. It follows that one obtains the Chebyshev fit P to f out of V over D by maximizing ρ over D^{n+1}. The relaxation process for maximizing the function ρ always converges to the correct answer. One thus has an algorithm for calculating the polynomial P to any degree of precision, the calculation being reduced to that of calculating the maximum of a function of one real variable. A Lipschitz condition is exhibited for ρ. (Received March 4, 1958.)

546-35. J. R. Shoenfield: *Constructive ordinals in formal systems*. Preliminary report.

To an axiom system for number theory, add a new predicate O, where O(x) is intended to mean that x is the number of a constructive ordinal (in the sense of Church-Kleene). Add axioms corresponding to the classes in the Church-Kleene definition. Theorem 1: If O(x) is provable in the above system, then the ordinal of x is less than ϵ_0. Theorem 2: If α is an ordinal between ω and ϵ_0, there are numbers x and y of α such that O(x) is provable and O(y) is not provable. Theorem 3: If A is a true sentence of number theory, then there is a number x of an ordinal less than ω^{ω} such that A is provable from O(x) and conversely. Theorem 4: If A is a sentence not containing O, then A is provable in the above axiom system if and only if it is provable in number theory. (Received March 4, 1958.)

546-36. I. S. Gál: *Remarks on* (m,n)-*compact spaces*.

The notions of (m,n)-compactness and of complete (m,n)-compactness were introduced by the author in a recent paper (*On a generalized notion of compactness* I - II, Proc. Koninkl. Akad. v. Wetenssch. vol. 60 (1957) pp. 421-435). (m,n)-compactness is a special case of a definition introduced by P. S. Alexandroff and P. S. Urysohn in 1924 (*On compact topological spaces*, Trudy Mat. Inst. Steklov vol. 31 (1950) p. 95), namely roughly X is (m,n)-compact if and only if it is [m + 1, n + 1] compact in the Alexandroff Urysohn sense. We have the following new results: A topological space X is (m,m + 1)-compact if and only if every set $S \subseteq X$ of cardinality card $S > m$ has an m-*accumulation* point in X. This leads to a simplified proof of Theorems 3 and 5 of the above mentioned paper. If X is completely (m,n)-compact for some $n > m$ then X is completely (m, ∞)-compact. This is a corollary of Theorem 4 but it can also be proved directly. It permits a reinterpretation of the conditions stated in Theorem 4. Theorem 8 was proved independently by Yu. M. Smirnov who showed that the product of a $(1,\infty)$-compact space and of an (m,n)-compact space is (m,n)-compact (*On the theory of finally compact spaces*, Ukrain. Mat. Žurnal vol. 3 (1951) pp. 52-60). (Received March 4, 1958.)

546-37. P. J. Cohen: *On Green's theorem*.

Let P(x,y) and Q(x,y) be functions continuous on the closure of a rectangle R. Assume further that $\partial P/\partial x, \partial P/\partial y, \partial Q/\partial x, \partial Q/\partial y$ exist everywhere in the interior of R, and that $\partial Q/\partial x - \partial P/\partial y$ is integrable in R. Then $\int_R(\partial Q/\partial x - \partial P/\partial y)\, dxdy = \int_{\partial R} Pdx + Qdy$, where ∂R denotes the positively oriented boundary of R. This extends results of Bochner and V. L. Shapiro who both assumed additional regularity of the functions P and Q. The method of proof is modeled after the proof of the Looman-Mensov theorem. The result extends to n-dimensions and more general regions. (Received March 5, 1958.)

546-38. W. F. Eberlein: *Integrals over function space*. III.

The fundamental quadrature problem for the interval (-1,1) is that of approximating the functional $I(x) = \int_{-1}^{1} x(t)dt$ by an N point quadrature functional $J(x) = \sum_0^N A_m\, x(t_m)$, N being assigned in advance, so that I - J is a "minimum." As the admissible class of functions x take the class e_1 [See Abstract 675t, 1957]. Then $I(x) - J(x) = \sum_0^\infty x_n e_n \equiv \epsilon(x)$, where e_n is the error at t^n. The "probable error" is then minimized by minimizing the mean-square error $\sigma^2(I - J) \equiv E(t^2)$, where E is the natural interval over the unit sphere S of e_1. It turns out that this procedure exhibits the classical polynomial procedures as local first approximations. (Received March 5, 1958.)

546-39. R. A. Kunze and E. M. Stein: *Some unusual aspects of harmonic analysis on the group of* 2×2 *real unimodular matrices*.

If G is the real line and f is a function in $L_1(G)$ the operation L_f of convolution by f on $L_2(G)$ is a bounded operator; however if f belongs to L_p where $p > 1$, L_f need not be a bounded operator. In contrast to this situation, it is shown that when G is the 2 × 2 real unimodular group that L_f is a bounded operator on $L_2(G)$ for $1 \leq p < 2$. The result is proved by a detailed analysis of the Fourier transform for this group. It is first shown that the Fourier transform of a function in L_p, $p < 2$ can be extended to be analytic in a suitable strip of representations (which include the continuous principal series). Next, convexity properties of analytic functions are used to show that the Fourier transform is uniformly bounded on the principal series. From this the result stated above follows. (Received March 5, 1958.)

546-40. W. T. Kyner: *An existence and uniqueness theorem for a nonlinear Stefan problem*.

Stefan problems are associated with the physical processes of change of state, e.g., melting of ice, recrystallization of metals, evaporation and condesensation, flow through porous media. These processes and their corresponding parabolic boundary value problems are characterized by the presence of a free boundary, e.g. the surface of the melting ice, which is not known in advance, but is to be determined as part of the problem. The purpose of the present paper is to prove that there exists a unique solution to a Stefan problem in which the density and thermal conductivity can vary with time, position, and temperature. The diffusion equation is therefore nonlinear. This problem specializes to those studied by Douglas, Datzeff, Evans, and Sestini. (Received March 5, 1958.)

546-41. D. E. Schroer: *Unified formulation of logical systems*.

A definition is given for the notion of a *combinatory logic* with finitely-many primitive symbols, finitely-many axioms, and finitely-many rules of a very simple type. Using well-known properties of the usual combinatory calculus, the following theorem is proved: Let S be any finite set of symbols and let S^* be the least set including S and closed under the operation of combination (binary juxtaposition followed by enclosure in parentheses); Let $T \subseteq S^*$. Then the following three conditions are equivalent: (1) for some (whence any) effective enumeration of S^*, the set of numbers of formulas in T is recursively enumerable; (2) there is a combinatory logic $\mathcal{L}$ and an $\mathcal{L}$-formula $\mathcal{J}$ such that T is $\mathcal{L}$-represented by $\mathcal{J}$ in the sense that for any $\mathcal{A}$, $\ulcorner(\mathcal{J}\mathcal{A})\urcorner$ is an $\mathcal{L}$-theorem iff $\mathcal{A} \in \mathcal{J}$; (3) there is a combinatory logic $\mathcal{L}$ such that the set L of $\mathcal{L}$-theorems is an S^*-conservative extension of T in the sense that

$T = S^* \cap L$. Since the set of wffs (or of all formulas) of any finitistic logical system is in effective one-to-one correspondence with a (decidable) subset of such an S^* (where S may be taken finite), the import of the above considerations is that every finitistic logical system can be formulated as a combinatory logic. Natural formulations exist in particular cases. (Received March 5, 1958.)

546-42. J. D. Stasheff: The "space of loops" isomorphism.

Let X,Y be spaces with base points. $\pi(X,Y)$ will denote the set of homotopy classes of maps $f\colon X \to Y$ (relative to base points). Theorem 1: If X,Y are CW complexes and if $\pi_i(X) = 0$ $i < n$ and $\pi_i(Y) = 0$ $i > 2n - 2$ then $\pi(X,Y)$ forms a group. If X' and Y' are CW complexes satisfying the same conditions as X and Y respectively, then maps $f\colon Y \to Y'$ and $g\colon X' \to X$ induce homomorphisms $f_{\#}\colon \pi(X,Y) \to \pi(X,Y')$ and $g^{\#}\colon \pi(X,Y) \to \pi(X',Y)$ respectively. ΩX, the "space of loops on X", will mean the functional space X^{S^1} of all maps of the circle into X (relative to base points). Given $f\colon X \to Y$, define $\Omega f\colon \Omega X \to \Omega Y$ by $((\Omega f)\lambda)(t) = f \circ \lambda(t)$ where $\lambda\colon S^1 \to X$ is a point of ΩX, $(\Omega f)\lambda$ is a point of ΩY. The correspondence $f \to \Omega f$ induces a function $\phi\colon \pi(X,Y) \to \pi(\Omega X, \Omega Y)$. Theorem 2: Under the same conditions on X,Y as for Theorem 1, the function ϕ is an isomorphism. The main tools used are elementary facts about obstruction theory and Postnikov systems, and the known case of Theorem 2 where Y is a $K(\pi,i)$-space with $i < 2n - 1$. (Received March 5, 1958.)

546-43. W. F. Stinespring: A sufficient condition for a integral operator to have a trace.

Let M be an n-dimensional manifold of class C^{r+1} where $r = [n/2]$. Let μ be a regular measure on M with compact support such that locally $d\mu$ is an n-form with a bounded measurable coefficient. Suppose that K is a function on $M \times M$ with continuous derivatives up through order r such that the rth derivatives satisfy Lipschitz conditions of order $> n/2 - r$. Then the operator on $L_2(M,\mu)$ with kernel K is of trace class. This theorem yields a sufficient condition for a function on a compact Lie group to have an absolutely convergent Fourier series. This condition generalizes Bernstein's theorem for ordinary Fourier series, and is sharper for compact groups than the results of the author for unimodular Lie groups presented in Bull. Amer. Math. Soc. Abstract 63-6-739. (Received March 5, 1958.)